Assignment #5

Due on Monday, September 19, 2011

Read Section 4.3 on *The Logarithm,* pp. 153–157, in *Essential Calculus with Applications* by Richard A. Silverman.

Read Section 4.2, *The Natural Logarithm Function,* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`, starting on page 22.

Background and Definitions

The natural logarithm function, $\ln\colon (0,\infty) \to \mathbf{R}$, is the unique solution to the initial value problem

$$\begin{cases} \dfrac{dy}{dt} = \dfrac{1}{t}; \\ y(1) = 0, \end{cases}$$

for $t > 0$, so that

$$\ln(t) = \int_1^t \frac{1}{\tau}\, d\tau, \qquad \text{for all } t > 0.$$

Do the following problems

1. Show that $\ln\left(\dfrac{a}{b}\right) = \ln a - \ln b$, for $a, b > 0$.

2. Let $f(t) = \ln\sqrt{1+t^2}$ for all $t \in \mathbf{R}$.

   (a) Compute $f'(t)$ and $f''(t)$.

   (b) Determine the intervals on the t–axis for which f is increasing or decreasing, and all local extrema; the values of t for which the graph of $y = f(t)$ is concave up, and those for which the graph is concave down; and all the inflection points of the graph of $y = f(t)$. Sketch the graph of $y = f(t)$.

3. Let $f(t) = t\ln t$ for $t > 0$.

   (a) Compute $f'(t)$ and $f''(t)$.

   (b) Determine the intervals on the t–axis for which f is increasing or decreasing, and all local extrema; the values of t for which the graph of $y = f(t)$ is concave up, and those for which the graph is concave down; and all the inflection points of the graph of $y = f(t)$. Sketch the graph of $y = f(t)$.

4. Evaluate the indefinite integral

$$\int \frac{1}{t+\sqrt{t}}\, dt$$

by making the change of variables $u = \sqrt{t}$.

5. Define $g(t) = t \ln t - t$ for all $t > 0$. Compute $g'(t)$ and use your result in order to obtain a formula for evaluating the indefinite integral

$$\int \ln u \; du.$$