Topics for Exam 1

1. The Fundamental Theorem of Calculus
   1.1 Solving the initial value problem $\begin{cases} \dfrac{dy}{dt} = f(t) \\ y(t_o) = y_o, \end{cases}$ where f is a continuous function defined on an interval containing t_o.
   1.2 Evaluating integrals: Changing variables
2. The natural logarithm and exponential functions
3. Solving first order differential equations
   3.1 Separation of variables
   3.2 Solving the linear first order equation with constant coefficients $\dfrac{dy}{dt} = ay + b$.
4. Applications to Modeling
   4.1 One–compartment models: conservation principle
   4.2 Models of population growth
5. Qualitative study of the first order differential equation: $\dfrac{dy}{dt} = g(y)$.
   5.1 Qualitative analysis of the logistic equation
   5.2 Qualitative analysis of linear first–order equations

Relevant sections in the text: 4.24, 4.4, 4.2, 4.3, 4.5, 5.2 and 5.1
Relevant chapters in the lecture notes: Chapters 2, 3 and 4

Important Concepts.
Differential equation, initial value problem, conservation principle

Important Results.

A conservation principle for a one-compartment model. *Let $Q(t)$ denote the amount of a substance in a compartment at time t. Then, the rate of the change of the substance in the compartment is determined by the differential equation:*

$$\frac{dQ}{dt} = \textit{Rate of substance in} - \textit{Rate of substance out},$$

where we are assuming that Q is a differentiable function of time

Existence and stability for the linear first–order differential equation $\dfrac{dy}{dt} = ay + b$, **where** $a \neq 0$**.** *The general solution of the equation is given by*

$$y(t) = \overline{y} + ce^{at},$$

where $\overline{y} = -\dfrac{b}{a}$ is the equilibrium solution to the equation. $\overline{y}$ is stable if $a < 0$, and unstable if $a > 0$.

Important Skills.

1. Know how to apply the conservation principle to derive differential equation models
2. Know how to use separation of variables to solve first order differential equations.
2. Know how to obtain qualitative information about solutions to first order differential equations.