Assignment #10

Due on Friday, November 2, 2012

Read Handout #2 on *The Real Numbers System Axioms.*

Read Section 4.6 on *Ordered Fields* on pp. 63–66 in Schramm's text.

Read Chapter 5 on *Upper Bounds and Suprema,* pp. 80–85, in Schramm's text.

Read Section 6.1 on *The Archimedean Property* , pp. 89–91, in Schramm's text.

Do the following problems

1. For real numbers a and b with $a < b$, let (a, b) denote the open interval from a to b:
$$(a, b) = \{x \in \mathbb{R} \mid a < x < b\}.$$
A subset, D, of the real numbers is said to be **dense** in $\mathbb{R}$ if and only if for every open interval, (a, b),
$$(a, b) \cap D \neq \emptyset;$$
that is, the intersection of any open interval with D is nonempty.

   Use the fact that between any two distinct real numbers there exists a rational number to prove that $\mathbb{Q}$ is dense in $\mathbb{R}$ according to the definition given above.

2. Show that $\mathbb{Z}$ is not dense in $\mathbb{R}$.

3. Let $a, b \in \mathbb{R}$ with $a < b$. Prove that the set $(a, b) \cap \mathbb{Q}$ is infinite.

4. Given sets A and B, the set of elements in A which are not in B is denoted by $A \setminus B$; that is,
$$A \setminus B = \{x \in A \mid x \notin B\}.$$
Thus, for instance, the set $\mathbb{R} \setminus \mathbb{Q}$ is the set of irrational numbers.

   Prove that $\mathbb{R} \setminus \mathbb{Q}$ is dense in $\mathbb{R}$.

5. Let $q \in \mathbb{Q}$ and α be an irrational number. Prove that
   (a) if $q \neq 0$, then $q\alpha$ is irrational, and
   (b) $q + \alpha$ is irrational for any $q \in \mathbb{Q}$.
   (c) What can you say about α^q?