

## Assignment #3

Due on Friday, September 21, 2012

**Read** Handout #2 on *The Real Numbers System Axioms*.

**Read** Section 3.1 on *The Rules of Arithmetic* on pp. 43–44 in Schramm’s text.

**Read** Section 3.2 on *Fields* on pp. 44–46 in Schramm’s text.

**Do** the following problems

Use the Field Axioms in Handout #2 to prove the following:

1. Let  $x$  denote a real number satisfying  $x^2 = x$ . Prove that either  $x = 0$  or  $x = 1$ .

(Note that  $x^2 = xx$ .)

2. Let  $a \in \mathbb{R}$ . Prove that if  $a \neq 0$ , then the equation

$$ax = b$$

has a unique solution for every  $b \in \mathbb{R}$ .

3. Let  $x \in \mathbb{R}$ . Prove that  $(-1)x$  is the additive inverse of  $x$ ; that is  $x + (-1)x = 0$ .

4. Prove that, for any real number,  $x$ ,

$$(-x)^2 = x^2.$$

5. Let  $a, b \in \mathbb{Q}$ , where  $a^2 + b^2 \neq 0$ .

(a) Explain by  $a^2 - 2b^2 \neq 0$ .

(b) Show that the multiplicative inverse of  $a + b\sqrt{2}$ , namely  $(a + b\sqrt{2})^{-1}$ , is of the form  $c + d\sqrt{2}$ , where  $c, d \in \mathbb{Q}$ .