Assignment #4

Due on Wednesday, September 26, 2012

Read Handout #2 on *The Real Numbers System Axioms.*

Read Section 4.6 on *Ordered Fields* on pp. 63–66 in Schramm's text.

Do the following problems Use the order and field axioms in Handout #2 to prove the following:

1. Let $a, b \in \mathbb{R}$. Prove that

$$a^2 + b^2 = 0 \text{ if and only if } a = 0 \text{ and } b = 0.$$

2. Use induction to prove that $n > 0$ for all $n \in \mathbb{N}$.

3. Let r be a rational number satisfying $r > 0$. Prove that there exists a rational number, q, such that

$$0 < q < r.$$

4. Let $a, b \in \mathbb{R}$. Suppose that $a < b + \varepsilon$ for every $\varepsilon > 0$. Prove that

$$a \leqslant b.$$

5. Let $x \in \mathbb{R}$. Prove that $0 \leqslant x < \varepsilon$ for every $\varepsilon > 0$ implies that $x = 0$.