Exam 1 (Part I)

Friday, October 12, 2012

Name: ________________________________

Provide complete arguments when asked to prove a statement in a question. You will be graded on how well you organize your proofs as well as the logical flow or your deductions. In this exam you'll be allowed to use the handout on the axioms of $\mathbb{R}$ (Handout #2). Use your own paper and/or the paper provided for you. Write you name on this page and staple it to your solutions. You have 50 minutes to work on the following 2 problems. Relax.

1. Provide concise answers to the following questions:

   (a) Give the negation of the following statement:

   "For every $\varepsilon > 0$, there exists $n_o \in \mathbb{N}$ such that

   $$n \geqslant n_o \Rightarrow |x_n - x| < \varepsilon."$$

   (b) Let A denote a subset of the real numbers. Give the negation of the statement:

   "A is bounded above."

   (c) Let a and b denote real numbers. Give the contrapositive of the statement:

   "If $ab = 0$, then either $a = 0$ or $b = 0$."

   (d) Let B denote a subset of the real numbers. Give the converse of the statement:

   "If B is non–empty and bounded below, then B has a greatest lower bound."

2. Let a denote a real number.

   (a) Use the field and order axioms of the real numbers to prove the statement:

   If $a > 1$, then $a^2 > 1$.

   (b) Use Mathematical Induction to prove the statement:

   If $a > 1$, then $a^n > 1$ for all $n \in \mathbb{N}$.