Review Problems for Exam #1

1. Let B denote a non–empty subset of the real numbers which is bounded below. Define
$$A = \{x \in \mathbb{R} \mid x \text{ is a lower bound for } B\}.$$
Prove that A is non–empty and bounded above, and that $\sup A = \inf B$.

2. Prove that, for any real number, x,
$$|x^2| = |x|^2 = x^2.$$

3. Let $a, b, c \in \mathbb{R}$ with $c > 0$. Show that $|a - b| < c$ if and only if $b - c < a < b + c$.

4. Let $a, b \in \mathbb{R}$. Show that if $a < x$ for all $x > b$, then $a \leqslant b$.

5. Show that the set $A = \{1/n \mid n \in \mathbb{N}\}$ is bounded above and below, and give its supremum and infimum.

6. Let $A = \{n + \frac{(-1)^n}{n} \mid n \in \mathbb{N}\}$. Compute $\sup A$ and $\inf A$, if they exist.

7. Let $A = \{1/n \mid n \in \mathbb{N} \text{ and } n \text{ is prime}\}$. Compute $\sup A$ and $\inf A$, if they exist.

8. Let A denote a subset of $\mathbb{R}$. Give the negation of the statement: "A is bounded above."

9. Let $A \subseteq \mathbb{R}$ be non–empty and bounded from above. Put $s = \sup A$. Prove that for every $n \in \mathbb{N}$ there exists $x_n \in A$ such that
$$s - \frac{1}{n} < x_n \leqslant s.$$

10. What can you say about a non–empty subset, A, of real numbers for which $\sup A = \inf A$.