Assignment #14

Due on Monday, November 26, 2012

Read Section 6.2, *Differentiable Functions,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Sections 2–1, 2–2, 2–3, 2–4 and 2–5, pp. 47–54, in *The Calculus Primer* by William L. Schaaf.

Background and Definitions

- *(Differentiable Functions).* Let f be a function defined on an open interval I and $t \in I$. We say that f is differentiable at t if the limit

$$\lim_{h \to 0} \frac{f(t+h) - f(t)}{h} \tag{1}$$

  exists. If the limit in (1) exists for all $t \in I$, we say that f is differentiable in I. If f is differentiable at t, the limit in (1) is called the **derivative** of f at t and is denoted by $f'(t)$ or $\dfrac{df}{dt}$.

- *(Some Properties of Differentiable Functions).* Let f and g be functions defined in an open interval, I. Assume that f and g are differentiable at some $t \in I$. Then,

  (i) The functions $f + g$ and $f - g$ are differentiable at t, and their derivatives at t are given by

$$(f+g)'(t) = f'(t) + g'(t) \quad \text{and} \quad (f-g)'(t) = f'(t) - g'(t),$$

  respectively;

  (ii) for any constant c, the function cf is differentiable at t, and

$$(cf)'(t) = cf'(t).$$

Do the following problems

1. Let f be a real valued function defined in an open interval, I.

   (a) Show that if f is differentiable at $a \in I$, then f is continuous at a.

   (b) Give and example of a real valued function that is continuous at a point, but it is not differentiable there.

2. Let $p(t) = a_n t^n + a_{n-1}t^{n-1} + \cdots + a_2 t^2 + a_1 t + a_o$, where $a_o, a_1, a_2, \ldots, a_n$ are real constants, and $t \in \mathbb{R}$.

   Use the properties of differentiable functions to deduce that the polynomial function p is differentiable in $\mathbb{R}$ and compute $f'(t)$ for all $t \in \mathbb{R}$.

3. Let $f(t) = \dfrac{1}{t}$, for $t \neq 0$. Compute the difference quotient $\dfrac{f(t+h) - f(t)}{h}$, for $h \neq 0$, and show that the limit

   $$\lim_{h\to 0} \frac{f(t+h) - f(t)}{h}$$

   exists, provided that $t \neq 0$.

   Deduce that f is differentiable at $t \neq 0$, and give a formula for computing $f'(t)$, for $t \neq 0$.

4. Let $f(t) = t + \dfrac{1}{t}$, for $t \neq 0$. Use properties of differentiable functions and the result of problem (3) to show that f is differentiable for $t \neq 0$ and compute $f'(t)$, for $t \neq 0$.

5. Let $f(t) = \sin t$ for all $t \in \mathbb{R}$.

   (a) Compute the difference quotient $\dfrac{f(0+h) - f(0)}{h}$, for $h \neq 0$, and show that its limit as $h \to 0$ exists.

   (b) Deduce that f is differentiable at 0 and compute $f'(0)$.