Assignment #3

Due on Wednesday, September 19, 2012

Read Section 3.2, *Limits of Functions,* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read on *The Limit Concept,* pp. 32–45, in *The Calculus Primer* by William L. Schaaf.

Background and Definitions

- *Radian Measure.* Figure 1 shows a sketch of a circle of radius R centered at the origin in the xy–plane $(x^2+y^2=R^2)$. A point moving along the circle starts at $(R,0)$ and moves in the counterclockwise sense along the circle with unit speed. Suppose that at some time t the point is at location P on the circle. Denote the

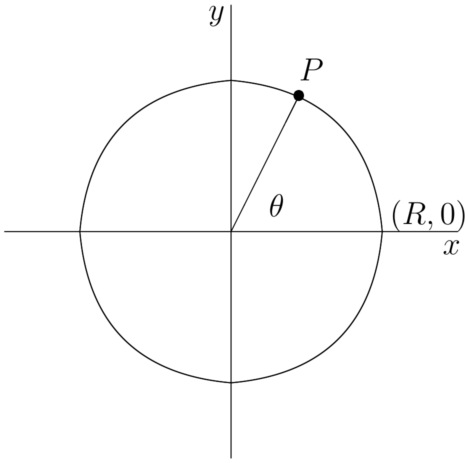

Figure 1: Unit Circle

distance traveled by the point by s. The radian measure of the angle, θ, made by the line segment from the origin to P with the positive x–axis is defined by

$$\theta = \frac{s}{R}, \quad \text{in radians.} \tag{1}$$

Note that since s and R have the same unit of length, the quantity θ defined in (1) is dimensionless; in other words, it is a real number.

- *The Sine and Cosine Functions.* Figure 2 shows the unit circle in the xy–plane, whose equation is given by $x^2+y^2=1$. Let t denote the radian measure of the angle that the segment from the origin to P makes with the positive x–axis.

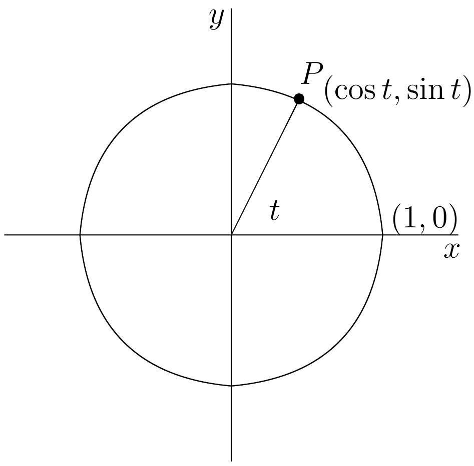

Figure 2: Graph of $x^2 + y^2 = R^2$

The trigonometric functions cos and sin give the cartesian coordinates of the point P; that is, P has coordinates $(\cos t, \sin t)$.

Do the following problems

1. *Radian Measure.* Given that the total distance traveled by the point P in Figure 1 when it goes once around the circle is $2\pi R$, compute the radian measure of the angle θ when P is at the points with cartesian coordinates $(R, 0)$, $(0, R)$, $(-R, 0)$, and $(0, -R)$.

2. *Sine and Cosine Functions.* Refer to the unit circle in Figure 2.

   (a) Give a justification to the trigonometric identity

   $$\cos^2 t + \sin^2 t = 1, \quad \text{for all } t.$$

   (b) Justify the statements

   $$\cos(-t) = \cos t, \quad \text{for all } t.$$

   and

   $$\sin(-t) = -\sin t, \quad \text{for all } t.$$

3. *Sine and Cosine Functions (continued).* Refer to the unit circle in Figure 2. Give the Cartesian coordinates of the points P for which the line segment from the origin to P makes an angle t wit the positive x–axis, where t is $\pi/6$, $\pi/4$ and $\pi/3$.

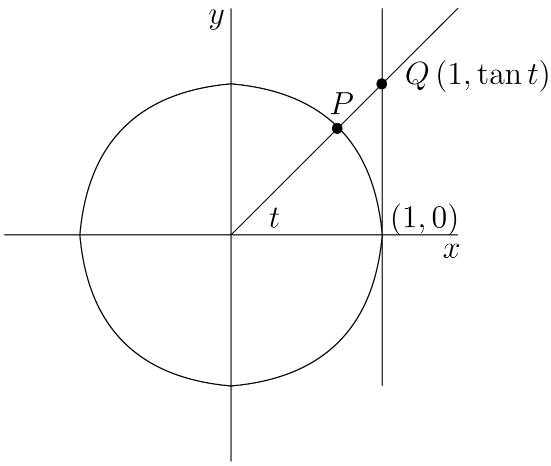

Figure 3: Unit Circle

4. *The Tangent Function.* Refer to the unit circle in Figure 3. The point labeled Q in Figure 3 is the intersection of the ray $\overrightarrow{OP}$ and the line tangent to the circle at $(1, 0)$. Let $(1, \tan t)$ denote the coordinates of Q.

   (a) Show that
   $$\tan t = \frac{\sin t}{\cos t}, \qquad \text{as long as } \cos t \neq 0. \tag{2}$$

   (b) Give the domain of definition of the tangent function, tan, given by (2).

5. *The Tangent Function (Continued).* Refer to the definition of the tangent function, tan, given by (2).

   Compute $\tan t$ for t being 0, $\pi/6$, $\pi/4$ and $\pi/3$ in radians.