Assignment #4

Due on Monday, September 24, 2012

Read Section 3.2, *Limits of Functions,* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read on *The Limit Concept,* pp. 32–45, in *The Calculus Primer* by William L. Schaaf.

Background and Definitions

A Few Limit Facts.

(i) $\lim_{t\to a} c = c.$

(ii) $\lim_{t\to a} t = a.$

(iii) $\lim_{t\to 0} \dfrac{\sin t}{t} = 1.$

(iv) $\lim_{t\to 0} \dfrac{\cos t - 1}{t} = 0.$

(v) $\lim_{t\to 0} \sin t = 0.$

(vi) $\lim_{t\to 0} \cos t = 1.$

Do the following problems

1. Use the limit fact $\lim_{t\to a} t = a,$ for all real numbers a, and the Function Limit Facts presented in the class lecture notes to compute the following limits

   (a) $\lim_{t\to a} t^2$ and $\lim_{t\to a} t^3.$

   (b) Give a formula for computing the limit $\lim_{t\to a} t^k,$ where k is any positive integer.

2. Use the limit results derived in Problem 1 and the limit facts presented in the class lecture notes to compute the following limits:

   (a) $\lim_{t\to a} ct^k,$ where c is any constant, and $k = 0, 1, 2, \ldots.$

(b) *Limits of Polynomial Functions.* A polynomial function, p, is given by an expression of the form

$$p(t) = c_o + c_1 t + c_2 t^2 + c_3 t^3 + \cdots + c_n t^n, \qquad \text{for } t \in \mathbf{R},$$

where $c_o, c_1, c_2, \ldots, c_n$ are real constants.
Compute $\lim_{t\to a} p(t)$ for any real number a.

3. Use the limit facts presented in the class lecture notes to compute the following limits:

   (a) $\lim_{t\to a} \frac{1}{t}$, for $a \neq 0$.

   (b) $\lim_{t\to a} \frac{1}{t-c}$, for $a \neq c$.

4. Compute the following limits:

   (a) $\lim_{t\to 2} \frac{t^3 - 3t^2 + 2t - 8}{t+1}$.

   (b) $\lim_{t\to 0} t\cos t$.

5. Use the fact that $\lim_{t\to 0} \frac{\sin t}{t} = 1$ and the limit facts presented in the class lecture notes to compute the following limits.

   (a) $\lim_{t\to 0} \frac{t}{\sin t}$.

   (b) $\lim_{t\to 0} \frac{t^2}{\sin t}$.