Assignment #9

Due on Friday, October 26, 2012

Read Section 5.2, *The Area Function,* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 5.3, *The Area Function as a Riemann Integral,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Sections 15-1, 15-2 and 15-3, pp. 318–321, in *The Calculus Primer* by William L. Schaaf.

Background and Definitions

- **Theorem** *(Some Properties of the Riemann Integral).* Let f and g denote piecewise continuous functions defined on an interval containing points a and b. Then,

  1. $\displaystyle\int_a^b cf(t)\ dt = c\int_a^b f(t)\ dt$, for any constant c.
  2. $\displaystyle\int_a^b [f(t)+g(t)]\ dt = \int_a^b f(t)\ dt + \int_a^b g(t)\ dt.$

- **Theorem** *(Some Integration facts).* Let a and b denote real numbers.

  1. $\displaystyle\int_a^b c\ dt = c(b-a)$, for any constant c.
  2. $\displaystyle\int_a^b t^m\ dt = \frac{1}{m+1}b^{m+1} - \frac{1}{m+1}a^{m+1}$ for $m = 1, 2, 3, \ldots$.

Do the following problems

1. Let f denote a piecewise continuous function defined on some interval I.

   Use properties of the area function to derive the following properties of the Riemann integral.

   (a) $\displaystyle\int_a^b f(t)\ dt = \int_a^c f(t)\ dt + \int_c^b f(t)\ dt$, for any point c in the interval I.

   (b) $\displaystyle\int_a^y f(t)\ dt - \int_a^x f(t)\ dt = \int_x^y f(t)\ dt$, for any points a, x and y in the interval I.

2. Let $f(t) = |t|$ for all $t \in \mathbb{R}$. Evaluate the definite integral $\displaystyle\int_{-1}^{2} f(t)\ dt$.

3. Let $f(t) = \begin{cases} 2 - t & \text{if } t < 2; \\ t - 1 & \text{if } t \geqslant 2. \end{cases}$

   Evaluate the definite integral $\displaystyle\int_{-1}^{2} f(t)\ dt$.

4. Let f denote the function defined by $f(t) = \sqrt{1 - t^2}$ for $-1 \leqslant t \leqslant 1$. Evaluate the definite integral $\displaystyle\int_{0}^{1/2} f(t)\ dt$.

5. Let f denote the polynomial function defined by $f(t) = t^4 - 2t^2 + 1$ for all $t \in \mathbb{R}$. Evaluate the definite integral $\displaystyle\int_{-1}^{1} f(t)\ dt$.