

Exam 1

Friday, October 5, 2012

Name: _____

Show all significant work and justify all your answers. This is a closed book exam. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 4 problems. Relax.

1. Suppose you are driving along the I-10 at a constant speed going east. You pass the Claremont exit at Indian Hill Boulevard (Exit 47) at 2:00 PM and the Vineyard exit in Ontario, about seven miles away, six minutes later.
 - (a) Express the distance you travel, $s(t)$ (in miles), from Exit 47 in terms of the time elapsed, t (in hours), from 2:00 PM.
 - (b) Sketch the graph of s as a function of t , where $s(t)$ is given by the formula found in part (a). Describe the graph of s .
2. Let a denote a real number satisfying $|a| < 1$ and put $c = \frac{|a|}{1 - |a|}$.
 - (a) Compute $\lim_{n \rightarrow \infty} \frac{c}{n}$. Explain your calculations.
 - (b) Use the inequality $|a|^n \leq \frac{c}{n}$, for $n = 1, 2, 3, \dots$, to compute $\lim_{n \rightarrow \infty} a^n$, for $|a| < 1$. Explain your calculations.
3. Let f denote a real valued function defined in some interval that contains a point a .
 - (a) State precisely what it means for f to be continuous at a .
 - (b) Let $f(t) = \begin{cases} \frac{t^2}{\sin t}, & \text{if } t \neq 0; \\ c, & \text{if } t = 0. \end{cases}$
Determine the value of c that will make the function f continuous at 0. Explain your answer and justify any assertion you make.
4. Let $f(t) = \begin{cases} 2 - t & \text{if } t < 2; \\ t - 1 & \text{if } t \geq 2. \end{cases}$
 - (a) Explain why f is continuous everywhere except at 2. Justify your answer.
 - (b) Describe the type of discontinuity that f has at 2 and explain why it cannot be removed by redefining f at 2.