## Review Problems for Exam 2

1. Let $f(t) = 0$ for $t < 0$, and $f(t) = 1 + t$ for $t \geqslant 0$, and let $A_f(0; x)$ denote the area under the graph of $f$ from 0 to $x$.

   (a) Give a formula for computing $A_f(0; x)$ for all values of $x$.

   (b) Sketch the graphs of $y = f(t)$ and $y = A_f(0; x)$.

2. Let $f(t) = \sqrt{t^4 + 1}$ for all $t \in \mathbb{R}$, and define $F(x) = \displaystyle\int_0^x f(t)\, dt$, for all $x \in \mathbb{R}$.

   (a) Explain why $F(x)$ increases as $x$ increases.

   (b) Determine the values of $x$ for which $F$ is negative and those for which $F$ is positive. Justify your answers.

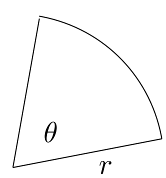
3. Let $f(t) = |t| + 1$ for all $t \in \mathbb{R}$. Sketch the graph of $y = f(x)$ and evaluate the area under the graph of $f$ from $-3$ to 3.

4. Let $f(t) = \sqrt{1 - (t-1)^2}$. Sketch the graph of $y = f(t)$ and evaluate the area under the graph of $f$ from 0 to 1 that lies above the $t$–axis.

5. Compute the area of the region in the $ty$–plane that lies below the line $y = t + 2$ and above the graph of $y = t^2$.

6. Find the area of the region under the graph of $y = \dfrac{1}{\sqrt{t}}$ and above the $t$–axis from $t = 1$ to $t = 4$.

7. The area, $A$, of the circular sector shown in the figure




   is given by the formula $A = \dfrac{1}{2}\theta r^2$, where $\theta$ is given in radians.

   Use this formula to evaluate the integral $\displaystyle\int_0^1 \sqrt{4 - t^2}\, dt$.

8. Let $f$ be a function defined by $f(t) = \begin{cases} 0, & \text{if } t < -1 \\ \sqrt{1 - t^2}, & \text{if } -1 \leqslant t < 0; \\ 1; & \text{if } t \geqslant 0. \end{cases}$

   Evaluate the area function $F(x) = \displaystyle\int_{-1}^x f(t)\, dt$, for all $x \in \mathbb{R}$, and sketch the graph of $y = F(x)$.