Topics for Exam 1

1. **Recovering a function from its rate of change:** The constant rate case
2. **The concept of limit**
   - 2.1 Limits of sequences of real numbers
   - 2.2 Limit of functions
3. **The concept of a continuous function**
   - 3.1 Definition of continuous functions
   - 3.2 Properties of continuous functions
   - 3.3 Discontinuous functions and types of discontinuity

Relevant sections in the online lecture notes: 2.1, 3.1, 3.2, 4.1 and 4.2.

Important Concepts: Constant rate of change; limit of a sequence; limit of a function; continuous functions; types of discontinuity.

Important Results

- *Constant Rate Functions.* Suppose that the rate of change of a function, f, is a constant, c, then $f(t) = f(t_o) + c(t - t_o)$, for all t.

- *The Squeeze Lemma for Sequences.* Let (a_n), (b_n) and (c_n) be three sequences. Suppose that there exists a positive integer n_1 such that

  $$a_n \leqslant b_n \leqslant c_n, \quad \text{for all } n \geqslant n_1.$$

  Assume in addition that the sequences (a_n) and (c_n) converge to the same limit ℓ; that is, $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} c_n = \ell$. Then, the sequence (b_n) converges to ℓ; that is, $\lim\limits_{n\to\infty} b_n = \ell$.

- *The Squeeze Lemma for Functions.* Let f, g and h denote a functions for which there exists a positive number δ such that

  $$f(t) \leqslant g(t) \leqslant h(t), \quad \text{for } 0 < |t - a| < \delta,$$

  and t is in the domains of f, g and h. Assume in addition that the limits of f and h as t approaches a exist and that $\lim\limits_{t\to a} f(t) = \lim\limits_{t\to a} h(t) = L$. Then, the limit of g as t approaches a exists and $\lim\limits_{t\to a} g(t) = L$.

Important Skills: Know how to recover a function from its rate of change when the rate is constant; know how to apply limit facts to compute limits of sequences; know how to apply function limit facts to compute limits of functions; know how to tell whether a given function is continuous or not.