

## Assignment #14

Due on Monday, October 28, 2013

**Read** Section 5.1 on the *Definition of the Joint Distribution* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

**Read** Section 5.2 on *Marginal Distributions* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

**Read** Section 3.4 on *Bivariate Distributions* in DeGroot and Schervish.

**Read** Section 3.5 on *Marginal Distributions* in DeGroot and Schervish.

**Do** the following problems

- Suppose that in an electric display sign there are three light bulbs in the first row and four light bulbs in the second row. Let  $X$  denote the number of bulbs in the first row that will be burned out at a specified time  $t$ , and let  $Y$  denote the number of bulbs in the second row that will be burned out at the same time  $t$ . Suppose that the joint pmf of  $X$  and  $Y$  is as specified in Table 1:

$X \setminus Y$	0	1	2	3	4
0	0.08	0.07	0.06	0.01	0.01
1	0.06	0.10	0.12	0.05	0.02
2	0.05	0.06	0.09	0.04	0.03
3	0.02	0.03	0.03	0.03	0.04

Table 1: Joint Probability Distribution for  $X$  and  $Y$ ,  $p_{(x,y)}$

Determine each of the following probabilities:

- (a)  $\Pr(X = 2)$    (b)  $\Pr(Y \geq 2)$    (c)  $\Pr(X \leq 2 \text{ and } Y \leq 2)$   
 (d)  $\Pr(X = Y)$    (e)  $\Pr(X > Y)$

- Suppose that  $X$  and  $Y$  have a continuous joint distribution for which the pdf is defined as follows:

$$f(x, y) = \begin{cases} cy^2 & \text{for } 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Determine

- (a) the value of  $c$ ; (b)  $\Pr(X + Y > 2)$ ; (c)  $\Pr(Y < 1/2)$ ;  
(d)  $\Pr(X \leq 1)$ ; (e)  $\Pr(X = 3Y)$ .

3. Suppose a point  $X$  is chosen at random from a region  $S$  in the  $xy$ -plane containing all points  $(x, y)$  such that  $x \geq 0$ ,  $y \geq 0$ , and  $4y + x \leq 4$ .

- (a) Determine the joint pdf of  $X$  and  $Y$ .  
(b) Suppose that  $S_o$  is a subset of the region  $S$  having area  $\alpha$ , and determine  $\Pr[(X, Y) \in S_o]$ .

4. Suppose that  $X$  and  $Y$  have a discrete distribution for which the joint pmf is defined as follows:

$$p_{(X,Y)}(x, y) = \begin{cases} \frac{1}{30}(x + y) & \text{for } x = 0, 1, 2 \text{ and } y = 0, 1, 2, 3, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Determine the marginal pmfs of  $X$  and  $Y$ .  
(b) Are  $X$  and  $Y$  independent?

5. Suppose the joint pdf of  $X$  and  $Y$  is as follows:

$$f_{(X,Y)}(x, y) = \begin{cases} \frac{15}{4}x^2 & \text{for } 0 \leq y \leq 1 - x^2 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Determine the marginal pdfs of  $X$  and  $Y$ .  
(b) Are  $X$  and  $Y$  independent?