Assignment #5

Due on Monday, September 23, 2013

Read Sections 2.5 and 2.6 on *Independent Events* and *Conditional Probability*, respectively, in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`
Read Section 2.1 on *The Definition of Conditional Probability* in DeGroot and Schervish.
Read Section 2.2 on *Independent Events* in DeGroot and Schervish.

Do the following problems

1. Let $(\mathcal{C}, \mathcal{B}, \Pr)$ be a probability space. Prove that if E_1 and E_2 are independent events in $\mathcal{B}$, then so are E_1 and E_2^c.

   Hint: Observe that $E_1 \setminus E_2$ is a subset of E_1.

2. Let A and B denote events in a probability space $(\mathcal{C}, \mathcal{B}, \Pr)$.

   (a) If $A \subseteq B$ with $\Pr(B) > 0$, what is the value of $\Pr(A \mid B)$?

   (b) If A and B are disjoint events and $\Pr(B) > 0$, what is the value of $\Pr(A \mid B)$?

3. A box contains r red balls and b blue balls. One ball is selected at random and the color is observed. The ball is then returned to the the box and k additional balls of the same color are also put in the box. A second ball is the selected at random, its color is observed, and it is returned to the box with k additional balls of the same color. Each time another ball is selected, the process is repeated. If four balls are selected, what is the probability that the first three balls will be red and the fourth one will be blue?

4. For any three events A, B and D, such that $\Pr(D) > 0$, prove that

$$\Pr(A \cup B \mid D) = \Pr(A \mid D) + \Pr(B \mid D) - \Pr(A \cap B \mid D).$$

5. *[The Monte Hall Problem].* In a game show, suppose there are three curtains. Behind one curtain is a nice prize while behind the other two there are worthless prizes. A contestant selects one curtain at random, and then Monte Hall (the game show host) opens one the other two curtains to reveal a worthless prize. Hall then expresses the willingness to trade the curtain that the contestant has selected for the other curtain that has not been opened. Should the contestant switch curtains or stick with the one that she has? If she sticks with the one she has then the probability of winning the prize is 1/3. Hence, to answer this question, you must determine the probability that she wins the prize given that she switches.