Solutions to Assignment #1

1. *(See Exercise 21 on page 33 in the text)* The dimensions of an olympic size swimming pool are 50 meters in length, 25 meters in width, and and average depth of about 2.5 meters.

   (a) Estimate the volume of water that the pool can hold in cubic feet.

   ***Solution**:* Denote the length, width, and average depth of the pool by ℓ, w and h, respectively. We then have that

$$
\begin{aligned}
\ell &= 50 \text{ m} \\
&= 50 \text{ m} \cdot \frac{3.281 \text{ ft}}{1 \text{ m}} \\
&\doteq 164 \text{ ft}; \\
w &= 2.5 \text{ m} \\
&= 2.5 \text{ m} \cdot \frac{3.281 \text{ ft}}{1 \text{ m}} \\
&\doteq 8.2 \text{ ft};
\end{aligned}
$$

   and

$$
\begin{aligned}
h &= 50 \text{ m} \\
&= 50 \text{ m} \cdot \frac{3.281 \text{ ft}}{1 \text{ m}} \\
&\doteq 164 \text{ ft}.
\end{aligned}
$$

   Then, an estimate for the volume of water in the pool is

$$
\begin{aligned}
V &= \ell w h \\
&\doteq (164)(82)(8.2) \text{ ft}^3,
\end{aligned}
$$

   or about $110,273.6$ cubic feet. □

   (b) Given that one cubic foot of water weighs about 62.4 pounds, estimate the weight of water in the pool in pounds.

Solution: The weight, W, of the water in the pool is about

$$\begin{aligned} W &\doteq 110,273.6 \text{ ft}^3 \cdot \frac{62.4 \text{ pound}}{1 \text{ ft}^3} \\ &\doteq 6.9 \times 10^6 \text{ pounds.} \end{aligned}$$

$\square$

2. *(See Exercise 22 on page 33 in the text)* The radius of the Earth is roughly $3,959$ miles.

   (a) Estimate the volume of the Earth in cubic meters.

   Solution: Denote the radius of the earth by R; we then have that

$$\begin{aligned} R &\doteq 3,959 \text{ miles} \cdot \frac{1609 \text{ m}}{1 \text{ mile}} \\ &\doteq 6.37 \times 10^6 \text{ m.} \end{aligned}$$

   Then, assuming that the earth is spherical, its volume is estimated by

$$\begin{aligned} V &= \frac{4}{3}\pi R^3 \\ &\doteq \frac{4}{3}\pi (6.37 \times 10^6)^3 \text{ m}^3 \\ &\doteq \frac{4}{3}\pi \cdot 258.47 \times 10^{18} \text{ m}^3 \\ &\doteq 1082.68 \times 10^{18} \text{ m}^3 \\ &\doteq 1.08 \times 10^{21} \text{ m}^3. \end{aligned}$$

$\square$

   (b) Given that mass of the earth is about 5.972×10^{24} kilograms, estimate the average density of the Earth in Kg/m^3. Give your answer also in grams per cubic centimeter.

   Solution: The density, D, of the earth is given by

$$D = \frac{M}{V},$$

where M is the mass of the earth. Using the estimate for the volume in the previous part, we then obtain that

$$\begin{aligned} D &\doteq \frac{5.972 \times 10^{24}\ \text{kg}}{1.08 \times 10^{21}\ \text{m}^3} \\ &\doteq 5.53 \times 10^3\ \text{kg/m}^3. \end{aligned}$$

To get the answer in grams per cubic centimeter, compute

$$\begin{aligned} D &\doteq 5.53 \times 10^3\ \frac{\text{kg}}{\text{m}^3} \cdot \frac{10^3\ \text{gr}}{1\ \text{kg}} \cdot \left(\frac{1\ \text{m}}{10^2\ \text{cm}}\right)^3 \\ &\doteq 5.53\ \text{gr/cm}^3. \end{aligned}$$

□

3. *(See Exercise 23 on page 33 in the text)* Seen from the air, much of Nebraska is covered with circular farm plots, each of about 1 mile in diameter.

   (a) Estimate the area of a farm plot in square feet.

   ***Solution**:* The area, A, of a circular plot of radius r is given by

$$A = \pi r^2.$$

   In this case the radius is 0.5 mile. Since there are 5280 feet in a mile, it follows that $r = 2640$ ft. We then have that the area of the circular plot is about

$$\begin{aligned} A &\doteq \pi (2640\ \text{ft})^2 \\ &\doteq 2.19 \times 10^7\ \text{ft}^2. \end{aligned}$$

□

   (b) Assuming that one square foot of the plot needs 1.31 gallons of irrigation water, estimate the total of amount of water needed to irrigate the entire plot.

   ***Solution**:* The total amount of water, W, needed to irrigate the plot is

$$W = 2.19 \times 10^7\ \text{ft}^2 \cdot \frac{1.31\ \text{gallons}}{1\ \text{ft}^2} = 2.87 \times 10^7\ \text{gallons}$$

□

4. *(See Exercise 26 on page 34 in the text)* The average diameter of a red blood cell is about 7 microns.

   (a) Assuming that a red blood cell is spherical, estimate its volume in cubic centimeters.

   ***Solution**:* Assuming the a red blood cell is spherical, its volume, v_c, is given by

$$v_c = \frac{4}{3}\pi r^3,$$

   where $r \doteq 3.5 \times 10^{-6}$ m, or $r \doteq 3.5 \times 10^{-4}$ cm. Thus, the volume of a single red blood cell is about

$$\begin{aligned} v_c &\doteq \frac{4}{3}\pi(3.5 \times 10^{-4} \text{ cm})^3 \\ &\doteq \frac{4}{3}\pi(42.875) \times 10^{-12}\text{cm}^3 \\ &\doteq 179.594 \times 10^{-12}\text{cm}^3 \\ &\doteq 1.8 \times 10^{-10}\text{cm}^3 \end{aligned}$$

   $\square$

   (b) Assuming that 45% of blood is made up of red blood cells, estimate the number of red blood cells in a pint of blood.

   ***Solution**:* One pint of blood is about 473 cubic centimeters. Forty five percent of that is the volume, V_R, of the red blood cells. Thus,

$$V_R \doteq (0.45)(473) \text{ cm}^3 \doteq 212.85 \text{ cm}^3.$$

   Since the volume of each red blood cell is about $v_c \doteq 1.8 \times 10^{-10}\text{cm}^3$, the number of red blood cells, N_R, is approximately

$$\begin{aligned} N_R &\doteq \frac{V_R}{v_c} \\ &\doteq \frac{212.85 \text{ cm}^3}{1.8 \times 10^{-10}\text{cm}^3} \\ &\doteq 118.25 \times 10^{10}, \end{aligned}$$

   or about 1.2×10^{12} red blood cells in a pint of blood. $\square$

5. *(See Exercises 27 and 28 on page 34 in the text)* Eighteen grams of distilled water contain approximately Avogadro's number of water molecules

   (a) Estimate the number of water molecules in one gram of distilled water.

   ***Solution**:* Since 18 grams of distilled water contain 6.02×10^{23} molecules, one gram of water contains, approximately,

   $$N \doteq 1 \text{ gr} \cdot \frac{6.02 \times 10^{23} \text{ molecules}}{18 \text{ gr}} \doteq 3.34 \times 10^{22} \text{ molecules.}$$

   □

   (b) How many grams would a trillion molecules of water weigh?

   ***Solution**:* A trillion molecules is 1×10^{12} molecules. Since 6.02×10^{23} have a mass of 18 grams, it follows that a trillion molecules of water have a mass, m, of, approximately,

   $$\begin{aligned} m &\doteq 1 \times 10^{12} \text{ molecules} \cdot \frac{18 \text{ gr}}{6.02 \times 10^{23} \text{ molecules}} \\ &\doteq 2.99 \times 10^{-11} \text{ gr.} \end{aligned}$$

   □