Assignment #13

Due on Friday, October 24, 2014

Read Section 4.3 on *Moments* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 4.4 on *Moments* in DeGroot and Schervish.

Do the following problems

1. Let $X \sim \text{Gemetric}(p)$, where $0 < p < 1$. Compute the mgf of X and use it to compute the $E(X)$, $E(X^2)$ and $\text{var}(X)$.

   Note: You will need the fact that

   $$\sum_{k=1}^{\infty} a^k = \frac{a}{1-a}, \quad \text{for } |a| < 1.$$

2. Let X have pdf given by

   $$f_X(x) = \begin{cases} \dfrac{1}{2}x^2 e^{-x}, & \text{if } x > 0; \\ \\ 0, & \text{if } x \leqslant 0. \end{cases}$$

   Compute the mgf of X and use it compute $E(X)$, $E(X^2)$ and $\text{var}(X)$.

3. Let X have mgf

   $$\psi_X(t) = (1-p)e^{-t} + pe^t, \quad \text{for all } t \in \mathbb{R},$$

   where $0 < p < 1$.

   (a) Give the distribution of X.

   (b) Use the mgf to find $E(X)$ and $\text{var}(X)$.

4. Let X have mgf

   $$\psi_X(t) = \frac{1}{6}e^{-2t} + \frac{1}{3}e^{-t} + \frac{1}{2}e^t, \quad \text{for all } t \in \mathbb{R}.$$

   Compute $\Pr(|X| \leqslant 1)$.

5. Suppose that X is a nonnegative random variable and that $\psi_X(\delta) < \infty$ for some $\delta > 0$. Show that $\psi_X(t)$ exists for all $t \in [0, \delta]$.