Assignment #3

Due on Wednesday, September 17, 2014

Read Section 2.4 on *Defining a Probability Function* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 1.5 on *The Definition of Probability* in DeGroot and Schervish.

Do the following problems

1. Consider two events A and B such that $\Pr(A) = 1/3$ and $\Pr(B) = 1/2$. Determine the value of $\Pr(B \cap A^c)$ for each of the following conditions:

   (a) A and B are disjoint;

   (b) $A \subseteq B$;

   (c) $\Pr(A \cap B) = 1/8$.

2. Consider two events A and B with $\Pr(A) = 0.4$ and $\Pr(B) = 0.7$. Determine the maximum and minimum possible values for $\Pr(A \cap B)$ and the conditions under which each of these values is attained.

3. Prove that for every two events A and B, the probability that exactly one of the two events will occur is given by the expression
$$\Pr(A) + \Pr(B) - 2\Pr(A \cap B).$$

4. Let A and B be elements in a σ–field $\mathcal{B}$ on a sample space $\mathcal{C}$, and let Pr denote a probability function defined on $\mathcal{B}$. Recall that $A \setminus B = \{x \in A \mid x \notin B\}$. Prove that if $B \subseteq A$, then
$$\Pr(A \setminus B) = \Pr(A) - \Pr(B).$$

5. Let $(\mathcal{C}, \mathcal{B}, \Pr)$ denote a probability space, and B an event in $\mathcal{B}$ with $\Pr(B) > 0$. Let
$$\mathcal{B}_B = \{D \subset \mathcal{C} \mid D = E \cap B \text{ for some } E \in \mathcal{B}\}.$$
We have already seen that $\mathcal{B}_B$ is a σ–field.

   Let $P_B \colon \mathcal{B}_B \to \mathbb{R}$ be defined by $P_B(A) = \dfrac{\Pr(A)}{\Pr(B)}$ for all $A \in \mathcal{B}_B$. Verify that $(B, \mathcal{B}_B, P_B)$ is a probability space; that is, show that $P_B \colon \mathcal{B}_B \to \mathbb{R}$ is a probability function.