Assignment #9

Due on Monday, October 13, 2014

Read Section 4.1 on *Expected Value of a Random Variable* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 4.1 on *The Expectation of a Random Variable* in DeGroot and Schervish.

Do the following problems

1. Two discrete random variable, X and Y, are said to be **independent** if
$$\Pr(X = x,\ Y = y) = \Pr(X = x) \cdot \Pr(Y = y)$$
for all possible values of x and y or X and Y, respectively.
Prove that if X and Y are discrete and independent, then
$$E(X + Y) = E(X) + E(Y).$$

2. Let X be a discrete random variable with pmf $p_X(x)$, and assume that $p_X(x)$ is positive at $x = -1, 0, 1$ and zero elsewhere.
   (a) If $p_X(0) = \dfrac{1}{4}$, find $E(X^2)$.
   (b) If $p_X(0) = \dfrac{1}{4}$ and if $E(X) = \dfrac{1}{4}$, determine $p_X(-1)$ and $p_X(1)$.

3. A bowl contains 10 chips, of which eight are marked \$2 and two are marked \$5 each. Let a person choose, at random and without replacement, three chips from the bowl. Assume the person is to receive the sum of the resulting amounts. On average, how much money will the person receive?

4. Let $p_X(k) = \left(\dfrac{1}{2}\right)^k$, for $k = 1, 2, 3, \ldots$, zero elsewhere, be the pmf of a discrete random variable X. Find the mean value of X.
Hint: For $|t| < 1$, define the function $f(t) = \displaystyle\sum_{k=0}^{\infty} t^k$. This is a geometric series which adds up to $\dfrac{1}{1-t}$. Compute $f'(t)$.

5. An experiment consists of tossing a balanced die until a 6 comes up. On average, how many tosses are required to get a 6? In other words, if X denotes the number of tosses it takes to get a 6, what is $E(X)$? Show your calculations and justify your reasoning.