Exam 1 (Part II)

Due on Monday, October 6, 2014

Name: ____________________

This is the out–of–class portion of Exam 1. There are three questions in this part of the exam. This is a closed–book and closed–notes exam; you may not consult with anyone. You may work on these questions as long as you wish. Show all significant work and give reasons for all your answers.

Students are expected to work individually on these problems. You may not consult with anyone.

I have read and agree to these instructions. Signature: ____________________

Please, write your name on this page and staple it to your solutions. Turn in your solutions at the start of class on **Monday, October 6, 2014.**

1. A point (x, y) is to be selected at random from a square S containing all the points (x, y) such that $0 \leqslant x \leqslant 1$ and $0 \leqslant y \leqslant 1$. Suppose that the probability that the selected point will belong to each specified subset of S is equal to the area of that subset, whenever that area of the subset is defined.

   Define the following events:

   $$\begin{aligned} E_1 &= \{(x,y) \in S \mid x \leqslant 0.5\}, \\ E_2 &= \{(x,y) \in S \mid y \geqslant 0.5\}, \\ \text{and} & \\ E_3 &= \{(x,y) \in S \mid x \leqslant 0.5,\ y \geqslant 0.5\} \cup \{(x,y) \in S \mid x \geqslant 0.5,\ y \leqslant 0.5\}. \end{aligned}$$

   (a) Compute $\Pr(E_1)$, $\Pr(E_2)$ and $\Pr(E_3)$.

   (b) Compute $\Pr(E_1 \cap E_2)$, $\Pr(E_1 \cap E_3)$ and $\Pr(E_2 \cap E_3)$.

   (c) Compute $\Pr(E_1 \cap E_2 \cap E_3)$.

   (d) Compute $\Pr(E_1 \mid E_2 \cap E_3)$.

   (e) Are the events E_1, E_2 and E_3 pairwise independent? Give reasons for your answer.

   (f) Are the events E_1, E_2 and E_3 mutually independent? Give reasons for your answer.

2. Suppose the probability density function (pdf) of a random variable, X, is as follows:

$$f_X(x) = \begin{cases} c\, e^{-x/\beta}, & \text{for } x \geqslant 0; \\ 0, & \text{elsewhere,} \end{cases}$$

where β is a given positive parameter.

(a) Find the value of c and sketch a graph of the pdf.

(b) Compute $\Pr(X > \beta)$.

(c) Find a positive value, m, for which

$$\Pr(X \leqslant m) = \frac{1}{2}.$$

3. Suppose that the time, T, that a manufacturing system is out of operation has cumulative distribution function (cdf) given by

$$F_T(t) = \begin{cases} 1 - \left(\dfrac{2}{t}\right)^2, & \text{for } t > 2; \\ \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Assume that t is measured in days. Estimate the probability that the system will be out of operation for at least 4 days.

(b) Assume that the resulting cost to the company is proportional to $Y = T^2$. Determine the probability density function (pdf) for Y.