Exam 2 (Part I)

Friday, November 7, 2014

Name: _____

This is the in–class portion of Exam 2. This is a closed–book and closed–notes exam; you may consult only the "Special Distributions" and the "Normal Distribution Probabilities Table" handouts.

Show all significant work and give reasons for all your answers. Use your own paper and/or the paper provided by the instructor. You have up to 40 minutes to work on the following 2 questions. Relax.

1. Let X denote a discrete random variable with possible values $x_1, x_2, \ldots, x_n$.

   (a) Let g denote a real valued function of a single variable. Give a formula for computing the expectation $E[g(X)]$.

   (b) An insurance policy pays \$100 per day for up to 3 days of hospitalization and \$50 per day for each day of hospitalization thereafter. The number of days of hospitalization, X, is a discrete random variable with probability mass function

$$
p_X(k) = \begin{cases} \dfrac{6-k}{15}, & \text{for } k = 1,2,3,4,5; \\[2mm] 0, & \text{elsewhere.} \end{cases}
$$

   Determine the expected payment for hospitalization under this policy.

2. Let X and Y denote independent random variables such that $X \sim \text{Normal}(\mu, 1)$ and $Y \sim \text{Normal}(\mu, 1)$, for some real parameter μ. Define $W = X - Y$.

   (a) Compute the moment generating function of W and use it to determine the distribution of W.

   (b) Estimate the probability $\Pr(|X - Y| < \sqrt{2})$. Explain the reasoning leading to your answer.