Exam 3 (Part I)

Friday, December 5, 2014

Name: ______________________________

This is the in–class portion of Exam 2. This is a closed–book and closed–notes exam; you may consult only the "Special Distributions" and the "Normal Distribution Probabilities Table" handouts.

Show all significant work and give reasons for all your answers. Use your own paper and/or the paper provided by the instructor. You have up to 40 minutes to work on the following 2 questions. Relax.

1. Let $X_1, X_2, X_3 \ldots$ denote a sequence of random variables.

   (a) State the Central Limit Theorem in the context of the sequence (X_k).

   (b) Let $X_1, X_2, \ldots, X_n$ denote random sample of size 49 from a Uniform$(0, 1)$ distribution. Let $\overline{X}_n$ denote the sample mean. Use the Central Limit Theorem to estimate the probability

   $$\Pr(0.4 < \overline{X}_n < 0.6).$$

2. Let X denote a random variable with mean μ and variance σ^2.

   (a) State the Chebyshev Inequality.

   (b) Let $X_1, X_2, \ldots, X_n$ denote random sample of size n from a distribution of mean μ and variance σ^2. Apply the Chebyshev inequality to get an upper bound for the probability

   $$\Pr(|\overline{X}_n - \mu| \geqslant k\sigma),$$

   where k is a positive number.

   (c) Let Y_n denote the number of heads in n tosses of a fair coin. Use the result from part (b) to obtain an upper bound for the the probability that Y_n deviates from $n/2$ by more than $5\sqrt{n}$.