## Assignment #1

**Due on Wednesday, September 10, 2014**

**Read** Chapter 1, *Motivation for the Course,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

**Read** Sections 2.1, 2.2, and 2.3 in Chapter 2, *Euclidean n–dimensional Space,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

**Read** Section 1.1 on *Vector Spaces* in Damiano and Little (pp. 1–10)

**Do** the following problems

1. Let $v_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

   (a) Write the vector $v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ as a linear combination of $v_1$ and $v_2$. That is, find scalars $c_1$ and $c_2$ such that $v = c_1 v_1 + c_2 v_2$.

   (b) Write any vector $v = \begin{pmatrix} x \\ y \end{pmatrix}$ in $\mathbb{R}^2$ as a linear combination of $v_1$ and $v_2$.

2. In this problem, $a$, $b$, $c$ and $d$ denote scalars, and elements in $\mathbb{R}^n$ are expressed as row vectors for convenience.

   (a) Find $a$, $b$ and $c$ so that $a(2,3,-1) + b(1,0,4) + c(-3,1,2) = (7,2,5)$, if possible.

   (b) Find $a$, $b$, $c$ and $d$ so that
   $$a(1,0,0,0,0) + b(1,1,0,0,0) + c(1,1,1,0,0) + d(1,1,1,1,0) = (8,5,-2,3,0),$$
   if possible.

3. Show that it is impossible to find scalars $a$, $b$, $c$ and $d$ so that
   $$a(1,0,0,0,0) + b(1,1,0,0,0) + c(1,1,1,0,0) + d(1,1,1,1,0) = (8,5,-2,3,1).$$

4. The equation $5x - 2y + 8z = 0$ describes a plane in $\mathbb{R}^3$. Let $(a_1, a_2, a_3)$ be any point on the plane; that is $5a_1 - 2a_2 + 8a_3 = 0$. Show that the vector $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ is a linear combination of the vectors $\begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix}$.

5. Let $W = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid (x_1, x_2, x_3) = c_1(2,5,0) + c_2(0,4,1)\}$ show that if $(x, y, z) \in W$, then $5x - 2y + 8z = 0$. What can you conclude from this and the statement in problem 4?