Assignment #6

Due on Friday, September 26, 2014

Read Section 2.7 on *Connections with the Theory of Systems of Linear Equations,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 1.5, *Interlude on Solving Systems of Linear Equations,* in Damiano and Little (pp. 32–45)

Do the following problems

1. Let W denote the solution space of the equation
$$3x_1 + 8x_2 + 2x_3 - x_4 + x_5 = 0.$$
Find a linearly independent subset, S, of $\mathbb{R}^5$ such that $W = \text{span}(S)$.

2. Let W denote the solution space of the system
$$\begin{cases} x_1 - 2x_2 - x_3 &= 0 \\ 2x_1 - 3x_2 + x_3 &= 0. \end{cases}$$
Find a linearly independent subset, S, of $\mathbb{R}^3$ such that $W = \text{span}(S)$.

3. In the following system, find the value or values of λ for which the system has nontrivial solutions. In each case, give a a linearly independent subset of $\mathbb{R}^2$ which generates the solution space.
$$\begin{cases} (\lambda - 3)x + \quad y &= 0 \\ \quad x + (\lambda - 3)y &= 0 \end{cases}$$

4. Let $v \in \mathbb{R}^n$ and S be a subset of $\mathbb{R}^n$.
   (a) Show that the set $\{v\}$ is linearly independent if and only if $v \neq \mathbf{0}$.
   (b) Show that if $\mathbf{0} \in S$, then S is linearly dependent.

5. Let v_1 and v_2 be vectors in $\mathbb{R}^n$, and let c be a scalar.
   (a) Show that $\{v_1, v_2\}$ is linearly independent if and only if $\{v_1, cv_1 + v_2\}$ is also linearly independent.
   (b) Show that
$$\text{span}(\{v_1, v_2\}) = \text{span}(\{v_1, cv_1 + v_2\}).$$