Assignment #7

Due on Monday, September 29, 2014

Read Section 2.7 on *Connections with the Theory of Systems of Linear Equations,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 1.5, *Interlude on Solving Systems of Linear Equations,* in Damiano and Little (pp. 32–45)

Do the following problems

1. Prove that if a homogeneous system of linear equations has one nontrivial solution, then it has infinitely many solutions.

2. Consider the vectors v_1, v_2, v_3 and v_4 in $\mathbb{R}^4$ given by
$$v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 2 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -1 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 0 \\ -1 \\ 3 \\ -5 \end{pmatrix}, \quad \text{and} \quad v_4 = \begin{pmatrix} 1 \\ -3 \\ 0 \\ 1 \end{pmatrix}.$$
Determine whether the set $\{v_1, v_2, v_3, v_4\}$ is linearly independent; if not, find a linearly independent subset of $\{v_1, v_2, v_3, v_4\}$ which spans $\operatorname{span}\{v_1, v_2, v_3, v_4\}$.

3. Let $W = \operatorname{span}\left(\left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix} \right\}\right)$. Find a linearly independent subset of W which spans W.

4. Let W denote the solution space of the system
$$\begin{cases} 3x_1 - 2x_2 - 2x_3 - x_4 + x_5 &= 0 \\ x_1 - 3x_2 - 2x_5 &= 0 \\ 2x_2 + x_3 + 2x_4 - x_5 &= 0 \\ -x_1 + x_2 - x_3 + x_4 - x_5 &= 0. \end{cases}$$
Find a linearly independent subset, S, of $\mathbb{R}^5$ such that $W = \operatorname{span}(S)$.

5. Determine whether or not the vector $\begin{pmatrix} 4 \\ 7 \\ 7 \\ 4 \end{pmatrix}$ lies in the span of the set
$$\left\{ \begin{pmatrix} 1 \\ 1 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 3 \\ -2 \end{pmatrix} \right\}.$$