

Assignment #8

Due on Friday, October 3, 2014

Read Section 2.8 on *Maximal Linearly Independent Subsets*, in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Read Section 1.2 on *Subspaces* in Damiano and Little (pp. 12–19)

Do the following problems

1. Given two subsets A and B of \mathbb{R}^n , the **union** of A and B , denoted by $A \cup B$, is the set which contains all vectors that are in either A or B ; in symbols,

$$A \cup B = \{v \in \mathbb{R}^n \mid v \in A \text{ or } v \in B\}.$$

- (a) Prove that $A \subseteq A \cup B$ and $B \subseteq A \cup B$.
 - (b) Suppose that W_1 and W_2 are two subspaces of \mathbb{R}^2 . Give an example that shows that $W_1 \cup W_2$ is not necessarily a subspace of \mathbb{R}^2 .
2. Given two subsets A and B of \mathbb{R}^n , the **sum** of A and B , denoted by $A + B$, is the set which contains all vectors sums, $v + w$, such that $v \in A$ and $w \in B$; in symbols,

$$A + B = \{u \in \mathbb{R}^n \mid u = v + w, \text{ where } v \in A \text{ and } w \in B\}.$$

Prove that if W_1 and W_2 are two subspaces of \mathbb{R}^n , then $W_1 + W_2$ is also a subspace of \mathbb{R}^n .

3. Let W_1 and W_2 be two subspaces of \mathbb{R}^n and define $W_1 + W_2$ as in the previous problem. Prove that $W_1 \cap W_2$, W_1 and W_2 are subspaces of $W_1 + W_2$.
4. Let W_1 and W_2 be two subspaces of \mathbb{R}^n and define $W_1 + W_2$ as in Problem 2 above. Suppose that $W_1 = \text{span}(S_1)$ and $W_2 = \text{span}(S_2)$, where $S_1 \subseteq W_1$ and $S_2 \subseteq W_2$. Prove that

$$W_1 + W_2 = \text{span}(S_1 \cup S_2).$$

5. Let S_1 and S_2 be two linearly independent subsets of \mathbb{R}^n . When can we say that $S_1 \cup S_2$ is linearly independent? Explain your reasoning.