Assignment #6

Due on Thursday, October 20, 2016

Read Section 5.1 on *Expected Value of a Random Variable* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 5.2 on *Properties of Expectation* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 5.3 on *Moments* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 5.4 on *Variance* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 4.1 on *The Expectation of a Random Variable* in DeGroot and Schervish.

Read Section 4.2 on *Properties of Expectations* in DeGroot and Schervish.

Read Section 4.3 on *Variance* in DeGroot and Schervish.

Read Section 5.2 on *The Bernoulli and Binomial Distributions* in DeGroot and Schervish.

Do the following problems.

1. A balanced die is tossed n times. Let X denote the number of 1's that come up. Give the pmf for X and compute its expectation.

2. Let X and Y denote independent Binomial(n, p) random variables and put $Z = X + Y$. Determine the pmf of Z and compute its expectation.

   Hint: Suppose there are n red balls and n blue balls in a box. Compute the number of ways of picking k balls out of the box, l of which are red and $k - l$ of which are blue.

3. *(Random Walk on the Integers).* A particle starts at $x = 0$ and, after one unit of time, it moves one unit to the right with probability p, for $0 < p < 1$, or to the left with probability $1 - p$. Let X_1 denote the position of the particle after one unit of time and X_2 denote that after 2 units of time. Give the probability mass functions for X_1 and X_2 and compute their expectations. Assume that at each time step, whether a particle will move to the right or to the left is independent of where it has been.

4. *(Random Walk on the Integers, Continued).* Let X_3 denote the position of the particle in the previous problem after 3 units of time. Give its pmf and expectation. Generalize this result to X_n, the position of the particle after n units of time.

5. Toss a coin 100 times, and let X denote the number of heads that come up. Given that the probability of a head is p, where $0 < p < 1$, give the distribution function of X and compute $\Pr(35 \leqslant X \leqslant 45)$ for the cases $p = 0.5$ and $p = 0.4$.

6. Let $X \sim \text{Uniform}(1, 2)$. Compute the variance of X.

7. Let $a \in \mathbb{R}$ and X be a discrete random variable with pmf

$$p_X(x) = \begin{cases} 1, & \text{if } x = a; \\ 0, & \text{elswhere.} \end{cases}$$

   Compute the variance of X.

8. Let X be a continuous random variable with variance σ^2. Define $Y = cX$, for some constant c. Compute the variance of Y in terms of σ^2.

9. Suppose that one word is selected at random from the sentence

   THE GIRL PUT ON HER BEAUTIFUL HAT.

   If X denotes the number of letters in the word that is selected, what is the value of $\text{var}(X)$?

10. Suppose that X is a random variable for which $E(X) = \mu$ and $\text{var}(X) = \sigma^2$. Show that

$$E[X(X-1)] = \mu(\mu - 1) + \sigma^2.$$