Assignment #7

Due on Thursday, October 27, 2016

Read Section 5.3 on *Moments* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 4.4 on *Moments* in DeGroot and Schervish.

Do the following problems.

1. Compute the moment generating function, $\psi_X(t)$, of a continuous random variable X with Uniform$(-1, 2)$ distribution. What should $\psi(0)$ be? Give also the second moment and variance of X.

2. Suppose that X is a random variable for which the mgf is as follows:
$$\psi_X(t) = e^{t^2+3t} \quad \text{for } -\infty < t < \infty.$$
Find the mean and variance of X.

3. Suppose that X is a random variable for which the mgf is as follows:
$$\psi_X(t) = \frac{1}{6}(4 + e^t + e^{-t}) \quad \text{for } -\infty < t < \infty.$$
Find the probability distribution of X.

4. Let X be a random variable with moment generating function (mgf) ψ_X.
   (a) Let $Y = cX$, where c is a constant. Compute the mgf of Y in terms of ψ_X.
   (b) Let $Y = X + a$, where a is a constant. Compute the mgf of Y in terms of ψ_X.

5. Let X be a random variable with moment generating function (mgf) ψ_X, expected value μ and variance σ^2. Put $Y = \dfrac{X-\mu}{\sigma}$
   (a) Compute the mgf of Y in terms of ψ_X.
   (b) Use the moment generating function found in part (a) to compute $E(Y)$ and var(Y).

6. Let $X \sim \text{Geometric}(p)$, where $0 < p < 1$. Compute the mgf of X and use it to compute the $E(X)$, $E(X^2)$ and $\text{var}(X)$.

   Note: You will need the fact that

$$\sum_{k=1}^{\infty} a^k = \frac{a}{1-a}, \qquad \text{for } |a| < 1.$$

7. Let X have pdf given by

$$f_X(x) = \begin{cases} \frac{1}{2} x^2 e^{-x}, & \text{if } x > 0; \\ \\ 0, & \text{if } x \leqslant 0. \end{cases}$$

   Compute the mgf of X and use it compute $E(X)$, $E(X^2)$ and $\text{var}(X)$.

8. Let X have mgf

$$\psi_X(t) = (1-p)e^{-t} + pe^t, \qquad \text{for all } t \in \mathbb{R},$$

   where $0 < p < 1$.

   (a) Give the distribution of X.

   (b) Use the mgf to find $E(X)$ and $\text{var}(X)$.

9. Let X have mgf

$$\psi_X(t) = \frac{1}{6}e^{-2t} + \frac{1}{3}e^{-t} + \frac{1}{2}e^t, \qquad \text{for all } t \in \mathbb{R}.$$

   Compute $\Pr(|X| \leqslant 1)$.

10. Suppose that X is a nonnegative random variable and that $\psi_X(\delta) < \infty$ for some $\delta > 0$. Show that $\psi_X(t)$ exists for all $t \in [0, \delta]$.