Review Problems for Exam 2

(1) A bowl contains 5 chips of the same size and shape. Two chips are red and the other three are blue. Draw three chips from the bowl at random, without replacement. Let X denote the number of blue chips in a drawing.

(a) Give the pmf of X.

(b) Compute $\Pr(X > 1)$.

(c) Compute $E(X)$.

(2) Let X have pmf given by $p_X(x) = \dfrac{1}{3}$ for $x = 1, 2, 3$ and $p(x) = 0$ elsewhere. Give the pmf of $Y = 2X + 1$.

(3) Let

$$f_X(x) = \begin{cases} \dfrac{1}{x^2} & \text{if } 1 < x < \infty; \\ 0 & \text{if } x \leq 1, \end{cases}$$

be the pdf of a random variable X. If E_1 denote the interval $(1, 2)$ and E_2 the interval $(4, 5)$, compute $\Pr(E_1)$, $\Pr(E_2)$, $\Pr(E_1 \cup E_2)$ and $\Pr(E_1 \cap E_2)$.

(4) A *mode* of a distribution of a random variable X is a value of x that maximizes the pdf or the pmf. If there is only one such value, it is called *the mode of the distribution.* Find the mode for each of the following distributions:

(a) $p(x) = \left(\dfrac{1}{2}\right)^x$, for $x = 1, 2, 3, \ldots$, and $p(x) = 0$, elsewhere.

(b) $f_X(x) = \begin{cases} 12x^2(1 - x), & \text{if } 0 < x < 1; \\ 0, & \text{elsewhere.} \end{cases}$

(5) Let X have pdf

$$f_X(x) = \begin{cases} 2x, & \text{if } 0 < x < 1; \\ 0, & \text{elsewhere.} \end{cases}$$

Compute the probability that X is at least $3/4$, given that X is at least $1/2$.

(6) Divide a segment at random into two parts. Find the probability that the largest segment is at least three times the shorter.

(7) Let X have pdf

$$f_X(x) = \begin{cases} x^2/9, & \text{if } 0 < x < 3; \\ 0, & \text{elsewhere.} \end{cases}$$

Find the pdf of $Y = X^3$.

(8) Assume that the random variable X has mgf

$$\psi_X(t) = \frac{e^t}{4 - 3e^t}, \qquad \text{for } t < \ln\left(\frac{4}{3}\right).$$

Compute the expected value, second moment and variance of X.

(9) Let X have mgf given by

$$\psi_X(t) = \frac{1}{3}e^t + \frac{2}{3}e^{2t}, \qquad \text{for } t \in \mathbf{R}.$$

(a) Give the distribution of X

(b) Compute the expected value and variance of X.

(10) Let X have mgf given by

$$f_X(x) = \begin{cases} \dfrac{e^t - e^{-t}}{2t}, & \text{if } t \neq 0; \\ 1, & \text{if } t = 0, \end{cases}$$

(a) Give the distribution of X

(b) Compute the expected value and variance of X.