

Review Problems for Final Exam

1. Three cards are in a bag. One card is red on both sides. Another card is white on both sides. The third card is red on one side and white on the other side. A card is picked at random and placed on a table. Compute the probability that if a given color is shown on top, the color on the other side is the same as that of the top.
2. An urn contains 10 balls: 4 red and 6 blue. A second urn contains 16 red balls and a number b of blue balls. A single ball is drawn from each urn. The probability that both balls are the same color is 0.44. Determine the value of b .
3. A blood test indicates the presence of a particular disease 95% of the time when the disease is actually present. The same test indicates the presence of the disease 0.5% of the time when the disease is not present. One percent of the population actually has the disease. Calculate the probability that a person has the disease given that the test indicates the presence of the disease.
4. A study is being conducted in which the health of two independent groups of ten policyholders is being monitored over a one-year period of time. Individual participants in the study drop out before the end of the study with probability 0.2 (independently of the other participants). What is the probability that at least 9 participants complete the study in one of the two groups, but not in both groups?
5. Suppose that $0 < \rho < 1$ and let $p(n) = \rho^n(1 - \rho)$ for $n = 0, 1, 2, 3, \dots$
 - (a) Verify that p is the probability mass function (pmf) for a random variable.
 - (b) Let X denote a discrete random variable with pmf p . Compute $\Pr(X > 1)$.
6. Let $N(t)$ denote the number of mutations in a bacterial colony that occur during the interval $[0, t]$. Assume that $N(t) \sim \text{Poisson}(\lambda t)$ where $\lambda > 0$ is a positive parameter.
 - (a) Give an interpretation for λ .
 - (b) Let T_1 denote the time that the first mutation occurs. Find the distribution of T_1 .
7. Let $X \sim \text{Exponential}(\beta)$, for $\beta > 0$. Compute the median of X .

8. A device runs until either of two components fails, at which point the device stops running. The joint density function of the lifetimes of the two components, both measured in hours, is

$$f(x, y) = \frac{x + y}{8}, \quad \text{for } 0 < x < 2 \text{ and } 0 < y < 2;$$

and 0 elsewhere.

What is the probability that the device fails during its first hour of operation?

9. A computer manufacturing company conducts acceptance sampling for incoming computer chips. After receiving a huge shipment of computer chips, the company randomly selects 800 chips. If three or fewer nonconforming chips are found, the entire lot is accepted without inspecting the remaining chips in the lot. If four or more chips are nonconforming, every chip in the entire lot is carefully inspected at the supplier's expense. Assume that the true proportion of nonconforming computer chips being supplied is 0.001. Estimate the probability the lot will be accepted.
10. Last month your company sold 10,000 new watches. Past experience indicates that the probability that a new watch will need repair during its warranty period is 0.002.
- Estimate the probability that no more than 5 watches will need warranty work. Explain the reasoning leading to your estimate.