Assignment #18

Due on Wednesday, November 30, 2016

Read Section 5.2, *The Principle of Linearized Stability,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`, starting on page 80.

Do the following problems.

1. For the following first–order differential equations, find all the equilibrium solutions and use the principle of linearized stability, when applicable, to determine whether the equilibrium solutions are asymptotically stable or unstable.

   (a) $\dfrac{dy}{dt} = y^2 - y - 6.$

   (b) $\dfrac{dy}{dt} = 0.5y(y-4)(y+2).$

2. For the following first–order differential equations, find all the equilibrium solutions and use the principle of linearized stability, when applicable, to determine whether the equilibrium solutions are asymptotically stable or unstable.

   (a) $\dfrac{dy}{dt} = (y-1)^2(y+2).$

   (b) $\dfrac{dy}{dt} = \dfrac{y^2-1}{y^2+1}.$

3. Give the linearization of the equation $\dfrac{dy}{dt} = \sin y$ around the equilibrium point $\overline{y} = \pi$. Solve the linearized equation and determine the long term behavior of its solutions as $t \to \infty$.

4. Determine whether the equilibrium solution $\overline{y} = 0$ is stable or unstable for the following equations.

   (a) $\dfrac{dy}{dt} = y^2$ (b) $\dfrac{dy}{dt} = -y^2$ (c) $\dfrac{dy}{dt} = y^3$ (b) $\dfrac{dy}{dt} = -y^3$

5. For which values of y_o does the IVP $\begin{cases} \dfrac{dy}{dt} &= y^3 - y \\ y(0) &= y_o \end{cases}$ have a solution, $y(t)$, which exists for all $t > 0$? For those values of y_o, what is $\lim\limits_{t\to\infty} y(t)$?