Assignment #3

Due on Monday, September 19, 2016

Read Section 4.1, *Recovering a Function from its Rate of Change,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 5.3 on *The Definite Integral,* pp. 369–376, in *Calculus for the Life Sciences* by Schreiber, Smith and Getz.

Background and Definitions

Let I denote an open interval of real numbers and $t_o \in I$. It was shown in the lecture notes that, if $f\colon I \to \mathbf{R}$ is a continuous real–valued function and $y_o \in \mathbf{R}$, then the function $y\colon I \to \mathbf{R}$ given by

$$y(t) = y_o + \int_{t_o}^{t} f(\tau)\, d\tau, \qquad \text{for all } t \in I, \tag{1}$$

is the unique solution to the initial value problem

$$\begin{cases} \dfrac{dy}{dt} = f(t) \\ y(t_o) = y_o. \end{cases} \tag{2}$$

In the first four problems of this assignments you will be asked to find solutions to the initial value problem in (2), for various examples of continuous functions, f, by using the formula in (1). Whenever it is possible, evaluate the integral on the right–hand side of (1).

Do the following problems

1. Solve the initial value problem

$$\begin{cases} \dfrac{dy}{dt} = t^2 \\ y(0) = 2. \end{cases}$$

2. Solve the initial value problem

$$\begin{cases} \dfrac{dy}{dt} = \sqrt{t} \\ y(1) = 0. \end{cases}$$

3. Let $y = y(t)$ denote the solution to the initial value problem

$$\begin{cases} \dfrac{dy}{dt} = \dfrac{1}{1+t^4} \\ y(0) = 0. \end{cases}$$

   (a) Use (1) to write down a formula for computing $y(t)$.

   (b) Compute $y'(t)$ and $y''(t)$.

   (c) Determine intervals on which (i) $y(t)$ increases, (ii) $y(t)$ decreases, (iii) the graph of $y = y(t)$ is concave up, and (iv) the graph of $y = y(t)$ is concave down.

   (d) Sketch the graph of $y = y(t)$.

4. Let $f(t) = \begin{cases} \dfrac{\sin t}{t} & \text{if } t \neq 0 \\ 1 & \text{if } t = 0. \end{cases}$

   (a) Explain why f is continuous at 0.

   (b) Use the fundamental Theorem of Calculus to write an expression for the solution to the initial value problem

$$\begin{cases} \dfrac{dy}{dt} = f(t) \\ y(0) = 0. \end{cases}$$

5. Define

$$F(t) = \int_0^{t^2} \frac{\sin(\tau)}{\tau}\, d\tau, \qquad \text{for } t \in \mathbf{R}.$$

   Use the Fundamental Theorem of Calculus and the Chain Rule to compute $F'(t)$.