Assignment #8

Due on Friday, October 7, 2016

Read Section 4.3 on *The Number e* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 4.4 on *The Exponential Function* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 1.4 on *Exponential Growth,* pp. 48–55, in *Calculus for the Life Sciences* by Schreiber, Smith and Getz.

Background and Definitions
The exponential function, $\exp\colon \mathbb{R} \to (0, \infty)$, given by $\exp(t) = e^t$, for all $t \in \mathbb{R}$, is the unique solution of the initial value problem

$$\begin{cases} \dfrac{dy}{dt} = y; \\ y(0) = 1. \end{cases}$$

Do the following problems

1. Use the properties of ln and exp to compute the exact value of $\ln(\sqrt{e})$. Compare your result with the approximation given by a calculator.

2. Let $f(t) = te^{-t^2}$ for all $t \in \mathbb{R}$. Compute $f'(t)$ and $f''(t)$. Determine the intervals on the t–axis for which f is increasing or decreasing, and all local extrema, the values of t for which the graph of f is concave up, and those for which the graph is concave down, and all the inflection points of the graph of f. Sketch the graph of $y = f(t)$.

3. Let $f(t) = te^{-t^2}$ for all $t \in \mathbb{R}$. For each $b > 0$ compute

$$F(b) = \int_0^b te^{-t^2}\, dt;$$

that is, $F(b)$ is the area under the graph of $y = f(t)$ from $t = 0$ to $t = b$.

Compute $\lim\limits_{b\to\infty} F(b)$. We denote this limit by $\displaystyle\int_0^\infty f(t)\, dt$, and call it the improper integral of f over the interval $(0, \infty)$.

4. Define $f(t) = t^t$, for all $t > 0$, and put $g(t) = \ln[f(t)]$ for all $t > 0$.

   (a) By differentiating g with respect to t, come up with a formula for computing $f'(t)$.

   Note: You will need to apply the Chain Rule when computing $\dfrac{d}{dt}[\ln[f(t)]]$.

   (b) Compute $f''(t)$. Does the graph of $y = f(t)$ have any inflection points?

5. Let t_o, r and y_o denote real numbers.

   Verify that $y(t) = y_o e^{r(t-t_o)}$, for $t \in \mathbb{R}$, is the unique solution of the initial value problem:

$$\begin{cases} \dfrac{dy}{dt} = ry; \\ y(t_o) = y_o. \end{cases}$$