

## Review Problems for Exam 1

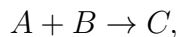
1. Water leaks out a barrel at a rate proportional to the square root of the depth of the water in the barrel at that time. Derive a differential equation for the depth,  $h(t)$ , of water in the barrel at time  $t$ , given that the cross-sectional area of the barrel is a constant  $A$ .
2. A compartment has a fixed volume,  $V$ , of isopropyl alcohol solution. A 75% solution of isopropyl alcohol is introduced into the compartment at a rate of  $F = 0.1$  liters per minute. Assume that the a well-stirred mixture of the solution flows out of the compartment at the same rate,  $F$ .
  - (a) Derive a differential equation for the concentration of alcohol, in percent volume, at any time  $t$ .
  - (b) Sketch possible solutions of the equation.
3. A 1500 gallon tank initially contains 600 gallons of water with 5 lbs of salt dissolved in it. Salt water enters the tank at a rate of 9 gal/hr with a salt concentration of 3 lbs/gal. If a well mixed solution leaves the tank at a rate of 6 gal/hr, derive a differential equation satisfied by the amount of salt in the solution in the tank.
4. Suppose that  $y = y(t)$  is a solution of the differential equation

$$\frac{dy}{dt} = -2ty \tag{1}$$

- (a) Assume that  $y(t) > 0$  for all  $t$ . Determine the values of  $t$  for which  $y(t)$  increases or decreases.
  - (b) Compute  $y''$  in terms of  $t$  and  $y$ , and determine the values of  $t$  for which the graph of  $y = y(t)$  is concave up or concave down.
  - (c) Given that  $y(0) = 1$ , use the qualitative information obtained in the previous parts to sketch the graph of  $y = y(t)$ .
5. Sketch possible solutions of the differential equation

$$\frac{dy}{dt} = (y - 1)(y - 2).$$

6. In a chemical reaction



let  $y(t)$  denote the concentration of the product  $C$  at time  $t$ . Assume that  $y$  is a differentiable function of  $t$ . If  $C_A$  denote the initial concentration of reactant  $A$  and  $C_B$  the initial concentration of reactant  $B$ , the Law of Mass Action states that

$$\frac{dy}{dt} = k(C_A - y)(C_B - y), \quad (2)$$

where  $k$  is a positive constant of proportionality.

Sketch possible solutions of (2) for the case in which  $C_A = 40$  and  $C_B = 80$ .

7. The following equation models the growth of a population that is being harvested at a constant rate:

$$\frac{dN}{dt} = 2N - 0.01N^2 - 75.$$

Sketch possible solutions of the differential equation.

8. Use the Chain Rule to show that  $y(t) = y_0 \exp(F(t))$ , where  $F$  is the antiderivative of  $f$  with  $F(0) = 0$ , is a solution of the initial value problem:  $\frac{dy}{dt} = f(t)y$ ,  $y(0) = y_0$ .
9. Evaluate the following integrals

$$(a) \int_0^1 \frac{e^{-x}}{2 - e^{-x}} dx \quad (b) \int \frac{1}{x \ln x} dx$$

$$(c) \int_1^2 \frac{\ln x}{x} dx \quad (d) \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

10. Solve the initial value problem

$$\begin{cases} \frac{dy}{dt} = \frac{1}{t + \sqrt{t}}; \\ y(1) = 2 \ln(2), \end{cases}$$

for  $t > 0$ .