Review Problems for Exam 2

1. Find a solution of the initial value problem $\dfrac{dy}{dt} = e^{t-y}, \quad y(0) = 1.$

2. The temperature in a hot iron decreases at a rate 0.11 times the difference between its present temperature and room temperature (20° C).

   (a) Write a differential equation for the temperature of the iron.

   (b) If the initial temperature of the rod is 100° C, and the time is measured in minutes, how long will it take for the rod to reach a temperature of 25°C?

3. Consider the first–order ordinary differential equation $\dfrac{dy}{dt} = y^2 - 2y + 1.$

   (a) Determine equilibrium points and determine the nature of the stability of the equilibrium solutions by means of the principle of linearized stability, if applicable.

   (b) Use separation of variables to find the general solution to the equation.

   (c) Use your result from the previous part to determine the nature of the stability of the equilibrium points.

   (d) Find a solution to the IVP $\begin{cases} \dfrac{dy}{dt} &= y^2 - 2y + 1; \\ y(0) &= 2, \end{cases}$ and determine its maximal interval of existence.

4. Solve the initial value problem $\dfrac{dy}{dt} = y + t^2, \qquad y(0) = 0,$ and compute $\lim_{t\to\infty} y(t)$.

5. **Logistic Growth with Harvesting.** The following differential equation models the growth of a population of size $N = N(t)$ that is being harvested at a rate proportional to the population density

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - EN, \tag{1}$$

   where r, K and E are non–negative parameters with $r > 0$ and $K > 0$.

   (a) Give an interpretation for this model. In particular, give interpretation for the term EN. The parameter E is usually called the harvesting *effort.*

   (b) Calculate the equilibrium points for the equation (1), and give conditions on the parameters that yield a biologically meaningful equilibrium point. Determine the nature of the stability of that equilibrium point. Sketch possible solutions to the equation in this situation.

(c) What does the model predict if $E \geqslant r$?