Review Problems for Final Exam

1. An initial population of $50,000$ inhabits a microcosm with carrying capacity of $100,000$. Suppose that, after five years, the population increases to $60,000$. Determine the intrinsic growth rate of the population.

2. Hydrocoden bitartrate is prescription drug used as a cough suppressant and pain reliever. Assume the drug is eliminated from the body by a natural decay process with half–life of 3.8 hours. The usual dose is 10 mg every 6 hours.

   (a) Use a conservation principle to derive a differential equation satisfied by the amount $Q(t)$ of the drug in the patient after a dose.

   (b) Assume that the amount of the drug in the patient prior to the dose is Q_o and that the drug is absorbed immediately. Give a formula for computing $Q(t)$, where t measures the length of time after the dose.

3. Suppose that alcohol is introduced into a 2-liter beaker, which initially contains distilled water, at a rate of 0.1 liners per minute. Assume that the a well–mixed mixture is removed from the beaker at the same rate.

   (a) Derive a differential equation for the concentration of alcohol in percent volume at any time t.

   (b) How long will it take for the concentration of alcohol to reach 50%?

4. A patient who has asthma is given a continuous infusion of theophylline to relax and open the air passages in the patient's lungs. The desired steady state level of theophylline in the patient's bloodstream is 15 milligrams per liter. The average half–life of theophylline is about four hours. Assume the bloodstream's volume in the patient is about 5.6 liters.

   (a) Determine the necessary infusion rate needed to maintain the theophylline level around 15 mg/L.

   (b) Determine how long it will take for the concentration of theophylline in the patient's body to reach 10 mg/L, assuming that there is no theophylline in the patient's body at time $t = 0$.

5. Estimate $\displaystyle\int_0^{0.047} e^{-t^2}\,dt$ and determine the accuracy of your estimate. Explain the reasoning leading to your answer.

6. Give the equilibrium points of the differential equation $\dfrac{dy}{dt} = (y+1)^2(y-1)$ and determine the nature of their stability. Sketch possible solutions.