

## Assignment #1

Due on Wednesday, September 6, 2017

Read Chapter 2, *An Example from Statistical Inference*, in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Read Section 1.4 on *Set Theory* in DeGroot and Schervish.

Do the following problems

1. Let  $\mathcal{C}$  denote a sample space and  $A$  be a subset of  $\mathcal{C}$ . Establish the following set theoretic identities, where  $\emptyset$  denotes the empty set. Justify your steps.

(a)  $A \cap \emptyset = \emptyset$ ,

(b)  $A \cup \emptyset = A$ .

2. Let  $\mathcal{C}$  denote a sample space and  $A$  and  $B$  denote subsets of  $\mathcal{C}$ . Establish the following set theoretic identities:

(a)  $(A^c)^c = A$ ,

(b)  $(A \cup B)^c = A^c \cap B^c$ ;

where  $A^c$  denote the complement of  $A$ .

3. Let  $\mathcal{C}$  denote a sample space and  $A$ ,  $B$  and  $C$  denote subsets of  $\mathcal{C}$ . Prove the following distributive properties:

(a)  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(b)  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

4. Let  $A$  and  $B$  be subsets of the sample space  $\mathcal{C}$ . The *set difference*  $A \setminus B$  is defined to be

$$A \setminus B = \{x \in A \mid x \notin B\};$$

thus,  $A \setminus B$  is a subset of  $A$  that contains those elements in  $A$  which are not in  $B$ .

Prove that

(a)  $A \setminus B = A \cap B^c$ ,

(b)  $B \setminus (A \cap B) = A^c \cap B$

5. Suppose that  $A \subseteq B$ . Prove that  $B^c \subseteq A^c$ .