

## Assignment #13

Due on Friday, November 10, 2017

Read Section 5.3 on *Moments* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Read Section 4.4 on *Moments* in DeGroot and Schervish.

Do the following problems

1. Suppose that  $X$  is a random variable for which the mgf is as follows:

$$\psi_X(t) = e^{t^2+3t} \quad \text{for } -\infty < t < \infty.$$

Find the mean and variance of  $X$ .

2. Suppose that  $X$  is a random variable for which the mgf is as follows:

$$\psi_X(t) = \frac{1}{6}(4 + e^t + e^{-t}) \quad \text{for } -\infty < t < \infty.$$

Find the probability distribution of  $X$ .

3. Let  $X$  be a random variable with moment generating function (mgf)  $\psi_X$ .

- (a) Let  $Y = cX$ , where  $c$  is a constant. Compute the mgf of  $Y$  in terms of  $\psi_X$ .
- (b) Let  $Y = X + a$ , where  $a$  is a constant. Compute the mgf of  $Y$  in terms of  $\psi_X$ .
- (c) Put  $Y = \frac{X - \mu}{\sigma}$ . Compute the mgf of  $Y$  in terms of  $\psi_X$ . Use this mgf to compute  $E(Y)$  and  $\text{Var}(Y)$ .

4. Let  $X$  have pdf given by

$$f_X(x) = \begin{cases} \frac{1}{2}x^2e^{-x}, & \text{if } x > 0; \\ 0, & \text{if } x \leq 0. \end{cases}$$

Compute the mgf of  $X$  and use it to compute  $E(X)$ ,  $E(X^2)$  and  $\text{Var}(X)$ .

5. Let  $X$  have mgf

$$\psi_X(t) = \frac{1}{6}e^{-2t} + \frac{1}{3}e^{-t} + \frac{1}{2}e^t, \quad \text{for all } t \in \mathbb{R}.$$

Compute  $\Pr(|X| \leq 1)$ .