

Assignment #4

Due on Monday, September 18, 2017

Read Section 3.4 on *Defining a Probability Function* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Read Section 1.5 on *The Definition of Probability* in DeGroot and Schervish.

Read Section 1.6 on *Finite Sample Spaces* in DeGroot and Schervish.

Do the following problems

1. Let $(\mathcal{C}, \mathcal{B}, \Pr)$ be a sample space. Suppose that E_1, E_2, E_3, \dots is a sequence of events in \mathcal{B} satisfying

$$E_1 \supseteq E_2 \supseteq E_3 \supseteq \dots$$

Prove that $\lim_{n \rightarrow \infty} \Pr(E_n) = \Pr\left(\bigcap_{k=1}^{\infty} E_k\right)$.

Hint: Use the analogous result for an increasing nested sequence of events presented in class and De Morgan's laws.

2. A point (x, y) is to be selected at random from a square S containing all the points (x, y) such that $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Suppose that the probability that the selected point will belong to each specified subset of S is equal to the area of that subset. Find the probability of each of the following subsets:

- (a) the subset of points such that $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 \geq \frac{1}{4}$;
- (b) the subset of points such that $\frac{1}{2} < x + y < \frac{3}{2}$;
- (c) the subset of points such that $y < 1 - x^2$;
- (d) the subset of points such that $x = y$.

3. In a random experiment, two balanced dice are rolled.

- (a) What is the probability that the sum of the two numbers that appear will be even?
- (b) What is the probability that the difference of the two numbers that appear will be less than 3?

4. A coin is tossed as many times as necessary to turn up one head. Thus, the elements of the sample space \mathcal{C} corresponding to this experiment are

$$H, TH, TTH, TTTH, \dots$$

Let \Pr be a function that assigns to these elements the values $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$ respectively.

- (a) Show that $\Pr(\mathcal{C}) = 1$.
- (b) Let E_1 denote the event $E_1 = \{H, TH, TTH, TTTH \text{ or } TTTTH\}$, and compute $\Pr(E_1)$.
- (c) Let $E_2 = \{TTTTH, TTTTTH\}$, and compute $\Pr(E_2)$, $\Pr(E_1 \cap E_2)$ and $\Pr(E_2 \setminus E_1)$
5. Let $\mathcal{C} = \{x \in \mathbb{R} \mid x > 0\}$ and define \Pr on open intervals (a, b) with $0 < a < b$ by

$$\Pr((a, b)) = \int_a^b e^{-x} dx.$$

- (a) Show that $\Pr(\mathcal{C}) = 1$.
- (b) Let $E = \{x \in \mathcal{C} \mid 4 < x < \infty\}$, and compute $\Pr(E)$, $\Pr(E^c)$ and $\Pr(E \cup E^c)$.