Topics for Exam 1

1. **Examples of Variational Problems**

   1.1 Minimal surfaces

   1.2 Geodesics

   1.3 Fastest descent

   1.4 Dirichlet integral

2. **Indirect Methods in the Calculus of Variations**

   2.1 Fundamental Lemmas

   2.2 Euler–Lagrange Equations

3. **Convex Minimization**

   3.1 Gâteaux differentiability

   3.2 Convex functionals

   3.3 Convex minimization theorem

Relevant sections in the online class notes: Sections 2.1, 2.2, 3.1, 3.2, 3.3, 4.1, 4.2, 4.3 and 4.4.

Important Concepts: Gâteaux differentiability, convex functionals, strictly convex functionals, Euler–Lagrange equations.

Important Skills

1. Know how to apply the fundamental lemmas in the Calculus of Variations.

2. Know how to obtain Euler–Lagrange equations for functionals defined in terms of integrals.

3. Know how to tell whether a functional is Gâtaux differentiable.

4. Know how to compute Gâteaux derivatives of Gâtaux differentiable functionals.

5. Know how to determine whether functionals are convex or strictly convex.

6. Know how to apply the convex minimization theorem.