

Assignment #9

Due on Wednesday, October 30, 2019

Read Section 4.3 on *Differentiable Scalar Fields* in the class Lecture Notes at <http://pages.pomona.edu/~ajr04747/>.

Do the following problems

1. Let $U = \mathbb{R}^n \setminus \{\mathbf{0}\} = \{v \in \mathbb{R}^n \mid v \neq \mathbf{0}\}$ and define $f: \mathbb{R}^n \rightarrow \mathbb{R}$ by $f(v) = \|v\|$ for all $v \in \mathbb{R}^n$.
 - (a) Prove that f is differentiable on U .
 - (b) Prove that f is not differentiable at the origin in \mathbb{R}^n .
2. Let $f: \mathbb{R}^3 \rightarrow \mathbb{R}$ be given by $f(x, y, z) = x^2y + y^2z + z^2x$, for all $(x, y, z) \in \mathbb{R}^3$. Compute all the first partial derivatives of f and verify that

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} + z \frac{\partial f}{\partial z} = 3f.$$

3. Find the gradient of f for each of the following scalar fields:

- (a) $f(x, y, z) = xe^{yz}$,

- (b) $f(x, y, z) = 1/\sqrt{x^2 + y^2 + z^2}$, $(x, y, z) \neq (0, 0, 0)$.

4. Let $f(x, y) = \begin{cases} (x^2 + y^2) \sin\left(\frac{1}{x^2 + y^2}\right), & \text{if } (x, y) \neq (0, 0); \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$

- (a) Show that the partial derivatives of f with respect to x and y do exist at $(0, 0)$, and compute $\frac{\partial f}{\partial x}(0, 0)$ and $\frac{\partial f}{\partial y}(0, 0)$.

- (b) Show that the partial derivatives of f with respect to x and y are not continuous at $(0, 0)$.

5. Let f be as in the previous problem. Show that f is differentiable at $(0, 0)$, and compute $Df(0, 0)$.