Topics for Final Exam

1. **Euclidean Space**

   1.1 Definition of n–Dimensional Euclidean Space

   1.2 Spans, lines and planes

   1.3 Dot product and Euclidean norm

   1.4 Orthogonality and projections

   1.5 The cross product in $\mathbf{R}^3$

   1.6 The triple scalar product in $\mathbf{R}^3$

2. **Continuous Functions**

   2.1 Vector fields, scalar fields and paths

   2.2 Definition of continuous function

   2.3 Compositions of continuous functions

   2.4 Limits and continuity

3. **Differentiability**

   3.1 Definition of differentiability

   3.2 The derivative as a linear approximation

   3.3 Derivatives of vector valued functions

   3.4 Derivatives of scalar fields

      i. The gradient

      ii. Partial derivatives

      iii. Directional derivatives

   3.5 The Jacobian matrix of a differentiable function

   3.6 The derivative of a composition of functions: The Chain Rule

4. **Differentiable Paths**

   4.1 C^1 curves and parametrizations

   4.2 Simple C^1 curves.

   4.3 Piecewise C^1 simple curves

   4.4 Simple closed curves

   4.5 Arclength of a curve; arclength parameter

5. **Integrals on Curves**

   5.1 Re-parametrizations

   5.2 Path integrals

   5.3 Line integrals

   5.4 Flux across a closed curve in the plane

6. **The Fundamental Theorem of Calculus and Double Integrals**

   6.1 The differential of a scalar field

   6.2 The divergence of a vector field

   6.3 The Fundamental Theorem of Calculus

      i. Line integral of a gradient field

      ii. The divergence theorem

      iii. Evaluating double integrals

      iv. Green's theorem

Relevant chapters in the online class notes: Chapters 2, 3, 4 and 5.

Important Concepts: Euclidean space, dot product, orthogonal projections, cross product, continuous function, differentiability, the derivative map, partial derivatives, the gradient of a scalar field, C^1 curves, piecewise C^1 curves, simple curves, simple closed curves, parametrizations, re–parametrizations, arclength, path integral, line integral, flux, differential of a scalar field, divergence of a vector field, and double integrals.

Important Skills: Know how to compute projections; know how to find equations of lines and planes; know how to show whether a function is continuous or not; know how to show whether a function is differentiable or not; know how to compute partial derivatives, gradients and directional derivative of scalar fields; Know how to compute the Jacobian matrix of a differentiable map; know how to apply the Chain Rule; know how to evaluate the arclength of C^1 curves; know how to evaluate path integrals; know how to evaluate line integrals; know how to compute flux across a simple closed curve; know how to evaluate the differential of a scalar field; know how to evaluate the divergence of a vector field; know how to evaluate double integrals; know how to apply the Fundamental Theorem of Calculus.