Assignment #10

Due on Monday March 3, 2008

Read Section 4.1 on *The Expectation of a Random Variable,* pp. 181–188, in DeGroot and Schervish.

Do the following problems

1. An experiment consists of tossing a balanced die until a 6 comes up. On average, how many tosses are required to get a 6? In other words, if X denotes the number of tosses it takes to get a 6, what is $E(X)$? Show your calculations and justify your reasoning.

2. Two discrete random variable, X and Y, are said to be **independent** if
$$\Pr(X = x,\ Y = y) = \Pr(X = x)\cdot\Pr(X = y)$$
for all possible values of x and y or X and Y, respectively.

   Prove that if X and Y are discrete and independent, then
$$E(X + Y) = E(X) + E(Y).$$

3. Let X be a discrete random variable with pmf $p_{_X}(x)$, and assume that $p_{_X}(x)$ is positive at $x = -1, 0, 1$ and zero elsewhere.

   (a) If $p_{_X}(0) = \dfrac{1}{4}$, find $E(X^2)$.

   (b) If $p_{_X}(0) = \dfrac{1}{4}$ and if $E(X) = \dfrac{1}{4}$, determine $p_{_X}(-1)$ and $p_{_X}(1)$.

4. A bowl contains 10 chips, of which eight are marked \$2 and two are marked \$5 each. Let a person choose, at random and without replacement, three chips from the bowl. If the person is to receive the sum of the resulting amounts, find this expectation.

5. Let $p_{_X}(k) = \left(\dfrac{1}{2}\right)^k$, for $k = 1, 2, 3, \ldots$, zero elsewhere, be the pmf of a discrete random variable X. Find the mean value of X.

   Hint: For $|t| < 1$, define the function $f(t) = \displaystyle\sum_{k=0}^{\infty} t^k$. This is a geometric series which adds up to $\dfrac{1}{1-t}$. Compute $f'(t)$.