Assignment #15

Due on Wednesday March 26, 2008

Read Section 3.4 on *Bivariate Distributions,* pp. 118–126, in DeGroot and Schervish.

Read Section 3.5 on *Marginal Distributions,* pp. 128–135, in DeGroot and Schervish.

Do the following problems

1. Let $F_{(X,Y)}$ be the joint cdf of two random variables X and Y. For real constants $a < b$, $c < d$, show that

   $\Pr(a < X \leqslant b, c < Y \leqslant d) = F_{(X,Y)}(b,d) - F_{(X,Y)}(b,c) - F_{(X,Y)}(a,d) + F_{(X,Y)}(a,c).$

   Use this result to show that $F(x,y) = \begin{cases} 1 & \text{if } x + 2y \geqslant 1, \\ 0 & \text{otherwise,} \end{cases}$ cannot be the joint cdf of two random variables.

2. Let $g(t)$ denote a non–negative, integrable function of a single variable with the property that

   $$\int_0^\infty g(t) \ \mathrm{d}t = 1.$$

   Define

   $$f(x,y) = \begin{cases} \dfrac{2g(\sqrt{x^2+y^2})}{\pi\sqrt{x^2+y^2}} & \text{for } 0 < x < \infty,\ 0 < y < \infty, \\ \\ 0 \text{otherwise.} \end{cases}$$

   Show that $f(x,y)$ is a joint pdf for two random variables X and Y.

3. Let X and Y have joint pdf

   $$f_{(X,Y)}(x,y) = \begin{cases} e^{-x-y} & \text{for } 0 < x < \infty,\ 0 < y < \infty, \\ 0 & \text{otherwise.} \end{cases}$$

   Define $Z = X + Y$. Compute $\Pr(Z \leqslant z)$ for $0 < z < \infty$ and give the pdf of Z.

4. Let X and Y have joint pdf

   $$f_{(X,Y)}(x,y) = \begin{cases} 1 & \text{for } 0 < x < 1,\ 0 < y < 1, \\ 0 & \text{otherwise.} \end{cases}$$

   Find the cdf and pdf of the product $Z = XY$.

5. Exercise 11 on page 136 in the text.