Assignment #16

Due on Monday March 31, 2008

Read Section 3.5 on *Marginal Distributions,* pp. 128–135, in DeGroot and Schervish.

Read Section 5.6 on *The Normal Distribution,* pp. 268–280, in DeGroot and Schervish.

Do the following problems

1. Suppose X and Y are independent and let $g_1(X)$ and $g_2(Y)$ be functions for which $E(g_1(X)g_2(Y))$ exists. Show that

$$E(g_1(X)g_2(Y)) = E(g_1(X)) \cdot E(g_2(Y))$$

   Conclude therefore that if X and Y are independent and $E(|XY|)$ is finite, then

$$E(XY) = E(X) \cdot E(Y).$$

2. Suppose X and Y are independent random variables for which the moment generating functions exist on some common interval of values of t. Show that

$$\psi_{_{X+Y}}(t) = \psi_{_X}(t) \cdot \psi_{_Y}(t)$$

   for t is the given interval.

3. Suppose that $X \sim \text{Normal}(\mu, \sigma^2)$ and define $Y = \dfrac{X-\mu}{\sigma}$.

   Prove that $Y \sim \text{Normal}(0, 1)$

4. Let X_1 and X_2 denote independent, $\text{Normal}(0, \sigma^2)$ random variables, where $\sigma > 0$. Define the random variables

$$\overline{X} = \frac{X_1 + X_2}{2} \quad \text{and} \quad Y = \frac{(X_1 - X_2)^2}{2\sigma^2}.$$

   Determine the distributions of $\overline{X}$ and Y.

   Suggestion: To obtain the distribution for Y, first show that

$$\frac{X_1 - X_2}{\sqrt{2}\,\sigma} \sim \text{Normal}(0, 1).$$

5. Let X_1, X_2, $\overline{X}$ and Y be as in the previous problem. Prove that $\overline{X}$ and Y are independent.

   Suggestion: Start working on this problem as soon as possible!