Assignment #20

Due on Monday April 21, 2008

Read Section 5.7 on *The Central Limit Theorem,* pp. 282–290, in DeGroot and Schervish.

Do the following problems

1. Let $X_1, X_2, X_3, \ldots$ denote a sequence of independent, identically distributed random variables with mean μ. Assume that the moment generating function of X_1 exists in some interval around 0. Use the mgf Convergence Theorem to show that the sample means, $\overline{X}_n$, converge in distribution to a limiting distribution with pmf

$$p(x) = \begin{cases} 1 & \text{if } x = \mu; \\ 0 & \text{elsewhere.} \end{cases}$$

2. Let $Y_n \sim \text{Binomial}(p, n)$, for $n = 1, 2, 3, \ldots$, and define $Z_n = \dfrac{Y_n - np}{\sqrt{np(1-p)}}$ for $n = 1, 2, 3, \ldots$ Use the Central Limit Theorem to find the limiting distribution of Z_n.

   Suggestion: Recall that Y_n is the sum of n independent Bernoulli(p) trials.

3. Exercise 2 on page 290 in the text

4. Exercise 4 on page 290 in the text

5. Exercise 12 on page 291 in the text