Assignment #4

Due on Wednesday February 6, 2008

Read Section 1.5 on *The Definition of Probability,* pp. 12–18, in DeGroot and Schervish.
Read Section 1.6 on *Finite Sample Spaces,* pp. 19–22, in DeGroot and Schervish.

Do the following problems

1. Let $(\mathcal{C}, \mathcal{B}, \Pr)$ be a sample space. Suppose that $E_1, E_2, E_3, \ldots$ is a sequence of events in $\mathcal{B}$ satisfying
$$E_1 \supseteq E_2 \supseteq E_3 \supseteq \cdots .$$
Then, $\displaystyle\lim_{n\to\infty} \Pr\,(E_n) = \Pr\left(\bigcap_{k=1}^{\infty} E_k\right)$.
Hint: Use the analogous result for an increasing nested sequence of events presented in class and De Morgan's laws.

2. Exercise 11 on page 18 in the text

3. Exercises 2 and 3 on page 22 in the text

4. A coin is tossed as many times as necessary to turn up one head. Thus, the elements of the sample space $\mathcal{C}$ corresponding to this experiment are
$$H, TH, TTH, TTTH, \ldots$$
Let Pr be a functions that assigns to these elements the values $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$ respectively.
   (a) Show that $\Pr(\mathcal{C}) = 1$.
   (b) Let E_1 denote the event $E_1 = \{H, TH, TTH, TTTH \text{ or } TTTTH\}$, and compute $\Pr(E_1)$.
   (c) Let $E_2 = \{TTTTH, TTTTTH\}$, and compute $\Pr(E_2)$, $\Pr(E_1 \cap E_2)$ and $\Pr(E_2 \setminus E_1)$

5. Let $\mathcal{C} = \{x \in \mathbb{R} \mid x > 0\}$ and define Pr on open intervals (a, b) with $0 < a < b$ by
$$\Pr((a, b)) = \int_a^b e^{-x} \ \mathrm{d}x.$$
   (a) Show that $\Pr(\mathcal{C}) = 1$.
   (b) Let $E = \{x \in \mathcal{C} \mid 4 < x < \infty\}$, and compute $\Pr(E)$, $\Pr(E^c)$ and $\Pr(E \cup E^c)$.