## Topics for Final Exam

1. **Euclidean Space**

   1.1 Definition of $n$–Dimensional Euclidean Space

   1.2 Spans, Lines and Planes

   1.3 Dot Product and Euclidean Norm

   1.4 Orthogonality and Projections

   1.5 The Cross Product in $\mathbf{R}^3$

2. **Functions**

   2.1 Vector fields, scalar fields and paths

   2.2 Definition of continuous function

   2.3 Compositions of Continuous Functions

   2.4 Limits and continuity

3. **Differentiability**

   3.1 Definition of differentiability

   3.2 The derivative as a linear approximation

   3.3 Derivatives of vector valued functions

   3.4 Derivatives of scalar fields

      3.4.1 The gradient

      3.4.2 Partial derivatives

      3.4.3 Directional derivatives

   3.5 Sufficient conditions for differentiability

      3.5.1 Differentiability of Paths; tangent line approximation.

      3.5.2 Differentiability of scalar fields.

      3.5.3 $C^1$ maps and differentiability.

      3.5.4 The Jacobain matrix

   3.6 Differentiability of Compositions: The Chain Rule

4. **Integration**

4.1 Path Integrals

4.1.1 Arc Length

4.1.2 Definition of the Path Integral

4.2 Line Integrals

4.2.1 Definition of the line integral

4.2.2 Gradient Fields

4.2.3 Flux Across Plane Curves

4.3 Differential Forms

4.3.1 Differential 1–forms

4.3.2 Differential 2–forms

4.3.3 The differential of a form

4.3.4 Calculus of Differential Forms

4.3.5 Evaluating 2–forms: Double Integrals

4.4 Fundamental Theorem of Calculus in $\mathbb{R}^2$

4.4.1 Green's Theorem

4.4.2 The Divergence Theorem in $\mathbb{R}^2$

**Relevant Sections in the Text:**

- Chapter 2 on *Vector Algebra*
- Section 7.1 on *Limits*
- Section 7.4 on *The Derivative*
- Section 7.3 on *Directional Derivatives*
- Section 3.3 on *Calculus of Curves*
- Section 7.6 on *The Chain Rule*
- Section 5.2 on *Line Integrals*
- Chapter 4 on *Differential Forms*
- Section 5.4 on *Multiple Integrals*
- Section 10.1 on *The Fundamental Theorem of Calculus*