Assignment #1

Due on Wednesday, January 28, 2009

Read Section 1.5 on *Euclidean Spaces* in Messer (pp. 21–27).

Do the following problems

1. Let $v_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

   (a) Write the vector $v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ as a linear combination of v_1 and v_2. That is, find scalars c_1 and c_2 such that $v = c_1 v_1 + c_2 v_2$.

   (b) Write any vector $v = \begin{pmatrix} x \\ y \end{pmatrix}$ in $\mathbb{R}^2$ as a linear combination of v_1 and v_2.

2. In this problem, a, b, c and d denote scalars, and elements in $\mathbb{R}^n$ are expressed as row vectors for convenience.

   (a) Find a, b and c so that $a(2, 3, -1) + b(1, 0, 4) + c(-3, 1, 2) = (7, 2, 5)$, if possible.

   (b) Find a, b, c and d so that

   $$a(1, 0, 0, 0, 0) + b(1, 1, 0, 0, 0) + c(1, 1, 1, 0, 0) + d(1, 1, 1, 1, 0) = (8, 5, -2, 3, 0),$$

   if possible.

3. Show that it is impossible to find scalars a, b, c and d so that

   $$a(1, 0, 0, 0, 0) + b(1, 1, 0, 0, 0) + c(1, 1, 1, 0, 0) + d(1, 1, 1, 1, 0) = (8, 5, -2, 3, 1).$$

4. The equation $5x - 2y + 8z = 0$ describes a plane in $\mathbb{R}^3$. Let (a_1, a_2, a_3) be any point on the plane; that is $5a_1 - 2a_2 + 8a_3 = 0$. Show that the vector $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ is a linear combination of the vectors

   $$\begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix}.$$

5. Let $W = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid (x_1, x_2, x_3) = c_1(2, 5, 0) + c_2(0, 4, 1)\}$ show that if $(x, y, z) \in W$, then $5x - 2y + 8z = 0$. What can you conclude from this and the statement in problem 4?